Prof. Dr. Stephan Held
Benjamin Klotz, M. Sc.

Exercise Set 2

Exercise 2.1. A Boolean function $f \in B_n$ *depends essentially* on all its variables if for every $1 \leq i \leq n$ the subfunctions $f|_{x_i=0}$ and $f|_{x_i=1}$ are different.

Let $f \in B_n$ be a function that essentially depends on all its variables. Show:

(a) $S_{B_2}(f) \geq n - 1$,

(b) $D_{B_2}(f) \geq \lceil \log_2 n \rceil$.

(5 points)

Exercise 2.2. Define a class of functions $(f_n)_{n \in \mathbb{N}}$ such that $f_n \in B_n$ and their SOP representations have size $\Omega(2^n)$.

(5 points)

Exercise 2.3. In this exercise we want to construct a B_2 circuit, based on the Kogge-Stone prefix graph, that computes c_{n+1}, c_i from x_i, y_i ($i = 1, \ldots, n \geq 4$). We consider its size, depth, and fanout as a B_2-circuit. E.g. the (B_2-)size of a single prefix gate (as presented in the lecture) is 3 while its (B_2-)depth is 2.

(a) What is the fanout of the Kogge-Stone prefix graph as presented in the lecture when implemented as a B_2-circuit.

(b) Show that the Kogge Stone prefix graph can be implemented as a B_2-circuit with depth $2\log(n) + 1$, size $\leq 4(n\log(n) - n/2)$, and fanout 2.

(c) Show that there exists an adder of depth $\leq 2\log(n) + 4\lceil \log(\log(n)) \rceil + 1$, size $\leq 10n$, and fanout 2.

(1 + 2 + 2 points)

Exercise 2.4. Let $f \in B_n$ be a Boolean function given as an oracle (i.e. for each $x \in \{0,1\}^n$ the value $f(x)$ can be computed in $\mathcal{O}(1)$ time). Show that the set $PI(f)$ of all prime implicants can be computed in $\mathcal{O}(n^2 3^n)$ time.

(5 points)

Deadline: April 18$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/chipss19.html`

In case of any questions feel free to contact me at klotz@or.uni-bonn.de.