Exercise Set 11

Exercise 11.1. The **Metric Bipartite Traveling Salesman Problem** is the problem of finding a Hamiltonian cycle of minimum cost in a bipartite graph $H$ with nonnegative cost function $d$ satisfying

$$d(a, b) + d(b, a') + d(a', b') \geq d(a, b')$$

for $\{a, b\}, \{a', b\}, \{a, b\}', \{a', b'\} \in E(G)$.

Prove that for any $k$, if there is a $k$-factor approximation algorithm for the **Metric Bipartite Traveling Salesman Problem**, there is also a $k$-factor approximation algorithm for the **Metric Traveling Salesman Problem**.

(4 points)

Exercise 11.2. Let $G$ be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for the TSP in this special case.

*Hint: You may use that a minimum weight 2-matching, i.e. a minimum weight subgraph of $G$ in which every vertex has degree 2, can be computed in polynomial time.*

(4 points)

Exercise 11.3. Consider the following algorithm for the **Symmetric Traveling Salesman Problem** with triangle inequality:

Start with an arbitrary vertex $u \in V(K_n)$. Find a shortest edge $e = \{u, v\} \in E(K_n)$ connecting $u$ to another vertex $v$. This yields a subtour $T = (u, v, u)$. Let $U := V(K_n) \setminus \{u, v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in $T$.

(ii) Add $w$ to $T$ between two neighbouring nodes $i, j \in T$ (by deleting the edge $\{i, j\}$ and connecting $i$ and $j$ with $w$), such that the cost of the new tour is minimized, i.e. find neighbouring $i, j \in T$ such that $c(i, w) + c(w, j) - c(i, j)$ is minimum. Remove $w$ from $U$ afterwards.

Show that this is a 2-approximation.

(4 points)
Exercise 11.4. Consider the following variant of the Metric TSP: Given an instance of the Metric TSP, we look for a Hamiltonian path of minimum weight (with arbitrary start- and endpoint). Give a $3/2$-approximation algorithm for this problem.

(4 points)

Deadline: Tuesday, June 25th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.