Exercise Set 9

Exercise 9.1. Consider the restriction \( P \) of the unweighted Vertex Cover Problem to graphs where the maximum degree of every vertex is bounded by a constant \( B \).

Let \( \varepsilon > 0 \). Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio \( 1 + \varepsilon \), then there exists a polynomial time approximation algorithm for problem \( P \) with performance ratio \( 1 + (B + 1)\varepsilon \).

(4 points)

Exercise 9.2. Let \( G = (V, E) \) be a graph with non-negative edge costs, and let \( S \subseteq V \) and \( R \subseteq V \) be disjoint vertex sets (“senders” and “receivers”). Consider the problem of finding a minimum cost subgraph of \( G \) that contains a path connecting each receiver to a sender.

(a) Prove that the restriction of this problem to instances with \( S \cup R = V \) is in \( P \).

(b) Prove that this problem is NP-hard and give a 2-factor approximation algorithm.

(2+2 points)

Exercise 9.3. Give an \( O(n^3t^2) \) algorithm for the Steiner Tree Problem in planar graphs with all terminals lying on the outer face, where \( n \) is the number of vertices and \( t \) the number of terminals.

(Hint: Modify the Dreyfus-Wagner algorithm.)

(4 points)
Exercise 9.4. Consider the following algorithm for the Steiner Tree Problem with 3 terminals $v_1$, $v_2$ and $v_3$: Find a shortest path $P$ between $v_1$ and $v_2$ and let $a$ be the distance of $v_3$ to $P$. Then find a vertex $z$ minimizing $\sum_{i=1}^{3} dist(v_i, z)$ under the conditions

(i) $dist(v_i, z) \leq dist(v_1, v_2)$ for $i \in \{1, 2\}$ and

(ii) $dist(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from $z$ to the terminals. Show that the algorithm needs $O(|E| + |V| \log(|V|))$ time and works correctly.

(4 points)

Deadline: Tuesday, June 4th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.