Exercise Set 6

Exercise 6.1. Recall the version of Knapsack from Exercise 5.3, where items can be used multiple times.

Give an FPTAS for this problem. (4 points)

Exercise 6.2. Let $A = (a_i)_{1 \leq i \leq p}$ and $B = (b_j)_{1 \leq j \leq q}$ be two inputs of the Bin Packing problem. We write $A \subseteq B$ if there are indices $1 \leq k_1 < k_2 < \cdots < k_p \leq q$ with $a_i \leq b_{k_i}$ for $1 \leq i \leq p$. An algorithm for the Bin Packing problem is called monotone if for inputs $A$ and $B$ with $A \subseteq B$ the algorithm needs at least as many bins for $B$ as for $A$. Prove or disprove:

(i) Next Fit is monotone.
(ii) First Fit is monotone. (2+2 points)

Exercise 6.3. Consider the Multiprocessor Scheduling Problem: Given a finite set $A$ of tasks, a processing time $t(a) \in \mathbb{R}_+$ for each $a \in A$ and a number $m$ of processors, find a partition $A = \bigcup_{i=1}^m A_i$ of $A$ such that $\max_{i=1}^m \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.

(i) Consider a greedy algorithm that successively assigns jobs (in an arbitrary order) to the currently least used machine. Show that this is a $2$-approximation algorithm.

(ii) Show that the modification of the greedy algorithm in which jobs are first sorted by $t(a)$ in non-increasing order and are then processed in that order is a $\frac{3}{2}$-approximation. (2+2 points)

Exercise 6.4. Show that the Bin-Packing Problem restricted to instances $a_1, \ldots, a_n$ with $a_i > \frac{1}{3}$ for $i = 1, \ldots, n$ can be solved in $O(n \log n)$ time. (4 points)

Information: The student council of mathematics will organize the math party on 9/05 in N8schicht. The presale will be held on Mon 6/05, Tue 7/05 and Wed 8/05 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de.
Deadline: Thursday, May 16th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.