Prof. Dr. Stefan Hougardy
Benjamin Rockel

Exercise Set 2

Exercise 2.1. For $k \in \mathbb{N}$ consider the following problem:

Instance: A set U and a set $\mathcal{S}$ of subsets of U with $|S| \leq k$ for all $S \in \mathcal{S}$, weights $w : U \to \mathbb{R}_{\geq 0}$.

Task: Find $T \subseteq U$ such that $T \cap S \neq \emptyset$ for each $S \in \mathcal{S}$ and $\sum_{t \in T} w(t)$ minimum.

(i) Show that this problem is NP-hard for $k \geq 2$.

(ii) Give a polynomial time k-factor approximation algorithm.

(iii) Give a linear time k-factor approximation algorithm for the special case that $w(t) = 1$ for $t \in U$.

(1+2+2 points)

Exercise 2.2. Show that the problem from exercise 1.2. is not strongly NP-hard if F and μ take only values in $\mathbb{N}$ and $F(e, \cdot)$ is nondecreasing for each $e \in E(G)$.

(3 points)

Exercise 2.3. Given a directed cycle $C = (V, E)$ and a set of undirected edges $E_1 \subseteq \{\{v, w\} | v, w \in V, v \neq w\}$. We are looking for an orientation $E_1^{\leftrightarrow}$ of E_1 such that in the digraph $G' = (V, E \cup E_1^{\leftrightarrow})$,

$$\max_{e \in E} |\{C' \text{ directed cycle} | e \in E(C') \text{ with } |E(C') \cap E_1^{\leftrightarrow}| = 1\}|$$

is minimum. Give a linear time 2-approximation algorithm for that problem.

(4 points)

Exercise 2.4. Consider the following algorithm for the optimization variant of the Simple Max Cut problem:

Given $G = (V, E)$ find a set $X \subseteq V$ maximizing $|\delta(X)|$. Start with $X = \emptyset$. Iteratively add a single vertex to X or delete a single vertex from X if this makes $|\delta(X)|$ larger. Stop, when no improvement is possible.

(i) Show that this algorithm is a polynomial time $\frac{1}{2}$-factor approximation algorithm.

(ii) Does the algorithm always find an optimum solution for planar graphs, or for bipartite graphs?

(2+2 points)

Prof. Dr. Stefan Hougardy
Benjamin Rockel

Deadline: Thursday, April 18$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.