Prof. Dr. Stefan Hougardy
Benjamin Rockel

Exercise Set 1

Exercise 1.1. Prove that SATISFIABILITY remains NP-complete if each clause contains at most three literals and each variable appears at most three times, but is in P if additionally each clause contains exactly three literals.

(4 points)

Exercise 1.2. Show that the following problem is NP-complete:

Instance: A directed graph G and two weight functions $F : E(G) \times \mathbb{Q}_{\geq 0} \to \mathbb{Q}_{\geq 0}$ and $\mu : E(G) \to \mathbb{Q}_{\geq 0}$, $D, L \in \mathbb{Q}$, $s, t \in V(G)$.

Given a path P in G, let $\{v_0, \dots, v_k\} = V(P)$ the vertices – and $\{e_1, \dots, e_k\} = E(P)$ the edges of P in the order of their appearance in P. We define the *load* of $v_i \in V(P)$ as

$$load_P(v_i) := \sum_{j=1}^{i} \mu(e_i)$$

and the *length* of P as

$$length(P) := \sum_{i=1}^{k} F(e_i, load_P(v_{i-1})).$$

Question: Is there a s-t-Path P such that $length(p) \leq D$ and $load(t) \leq L$?

(4 points)

Exercise 1.3. Show that the following problem is NP-complete:

Instance: A directed Graph G.

Question: Is there some $X \subseteq G$ such that $E(G[X]) = \emptyset$ and that for all $v \in V \setminus X$ we have $\delta^+_{G[X \cup \{v\}]}(v) \neq \emptyset$?

Hint: Use a reduction from SATISFIABILITY.

(4 points)

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Definition. For $\tau \leq 1$, a *τ-approximation algorithm* for the maximum stable set problem is a polynomial time algorithm that computes for every undirected graph $G = (V, E)$ a stable set $S \subseteq V$ such that $|S| \geq \tau \cdot \max\{|S^*| \,|\, S^* \subseteq V \text{ is a stable set}\}$.

Exercise 1.4. Prove: If there is a $\frac{1}{2}$-approximation algorithm for the maximum stable set problem, there is also a $(1 - \epsilon)$-approximation algorithm for every $\frac{1}{2} \geq \epsilon > 0$.

(4 points)

Deadline: Thursday, April 11$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.