Exercise Set 10

Exercise 10.1. Let $\alpha > 1$ and $1 \leq \beta < 1 + 2/\left(\alpha - 1\right)$. Construct a connected, planar graph $G$ with $w : E(G) \to \mathbb{R}_+$ and $r \in V(G)$ that contains no spanning tree $T$ with the following properties:

(a) For each $v \in V(G)$: $\text{dist}_{w,T}(r, v) \leq \alpha \cdot \text{dist}_{w,G}(r, v)$.

(b) For a minimum-spanning tree $M$: $\sum_{e \in E(T)} w(e) \leq \beta \cdot \sum_{e \in E(M)} w(e)$.

(5 points)

Exercise 10.2. Given a root $r \in \mathbb{R}^2$, a finite set of sinks $S \subset \mathbb{R}^2$, Lagrangean multipliers $(\lambda_s)_{s \in S}$, the rectilinear cost-distance Steiner arborescence problem asks for a Steiner arborescence $Y$ rooted at $r$, minimizing

$$
\sum_{(v, w) \in E(Y)} ||v - w||_1 + \sum_{s \in S} \lambda_s \cdot \left( \sum_{(v, w) \in E(Y[r, s])} ||v - w||_1 \right)
$$

Using the light-approximate shortest path tree algorithm, approximate this problem up to a factor of 3 in $O(n \log n)$.

(5 points)

Exercise 10.3. A posynomial function $f : \mathbb{R}^n_{>0} \to \mathbb{R}$ is of the form

$$
f(x) = \sum_{k=1}^{K} c_k \prod_{i=1}^{n} x_i^{a_{ik}}
$$

for $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$.

(a) Give an example for a non-convex posynomial function.

(b) Let $f$ be a posynomial function with lower and upper bounds $l, u \in \mathbb{R}^n_{>0}$, $l \leq u$ on the variables. Show that each local minimum of $f$ on the box $[l, u]$ is also a global minimum of $f$ on $[l, u]$.

Hint: Use a logarithmic variable transformation to derive an equivalent convex problem.
Exercise 10.4. Consider a chain of $n \in \mathbb{N}$ continuously sizable inverters with sizes $x_i > 0$ ($1 \leq i \leq n$) depicted in Figure 10.1. Assume that the delay $\theta_i$ through inverter $i$ is given by

$$\theta_i(x) = \alpha + \frac{\beta \cdot x_{i+1}}{x_i}$$

for $1 \leq i < n - 1$

where $x = (x_1, \ldots, x_n)$, $\alpha \geq 0$, $\beta > 0$. Wire delays, slews and transitions are ignored.

Derive a closed formula for the size $x_i$ of the $i$-th inverter in a solution $x$ of the total delay minimization problem for fixed $x_1, x_n$:

$$\min \left\{ \sum_{i=1}^{n-1} \theta_i(x) : x_i > 0 \text{ for all } 2 \leq i \leq n - 1 \right\}.$$

(5 points)

Deadline: July 10$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss18/chipss18.html

In case of any questions feel free to contact me at bihler@or.uni-bonn.de.