Exercise Set 5

Exercise 5.1.

(a) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which for any instance $I = (a_1, \ldots, a_n)$ of the Bin Packing problem finds a packing using the optimum number of bins but possibly violating the capacity constraints by $\varepsilon$, i.e. an $f : \{1, \ldots, n\} \to \{1, \ldots, \text{OPT} (I)\}$ with $\sum_{i \in j} a_i \leq 1 + \varepsilon$ for all $j \in \{1, \ldots, \text{OPT} (I)\}$.

*Hint:* Use Exercise 4.3.

(b) Use (a) to show that the **Multiprocessor Scheduling Problem** (see Exercise 4.2) has an approximation scheme.

(8 points)

Exercise 5.2. Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets (“senders” and “receivers”). Consider the problem of finding a minimum cost subgraph of $G$ that contains a path connecting each receiver to a sender.

(a) Prove that the restriction of this problem to instances with $S \cup R = V$ is in $P$.

(b) Prove that this problem is $NP$-hard and give a 2-factor approximation algorithm.

(4 points)

Exercise 5.3. Consider the **Directed Steiner Tree Problem**: Given a edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at $r$ that contains every vertex in $T$.

Show that a $k$-approximation algorithm for the **Directed Steiner Tree Problem** can be used to obtain a $k$-approximation algorithm for **Set Cover**.

(4 points)
Exercise 5.4. Consider the restriction \( \mathcal{P} \) of the unweighted Vertex Cover Problem to graphs where the maximum degree of every vertex is bounded by a constant \( B \).

Let \( \varepsilon > 0 \). Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio \( 1 + \varepsilon \), then there exists a polynomial time approximation algorithm for problem \( \mathcal{P} \) with performance ratio \( 1 + (B + 1)\varepsilon \).

(4 points)

Deadline: Tuesday, May 29th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss18/appr_ss18_ex.html

In case of any questions feel free to contact me at traub@or.uni-bonn.de.