Prof. Dr. Stephan Held
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Exercise Set 11

Exercise 11.1. Let $\alpha > 1$ and $1 \le \beta < 1 + 2/(\alpha - 1)$. Construct a connected, planar graph G with $w : E(G) \to \mathbb{R}_+$ and $r \in V(G)$ that contains no spanning tree T with the following properties:

(a) For each $v \in V(G)$: $\mathrm{dist}_{w,T}(r, v) \le \alpha \cdot \mathrm{dist}_{w,G}(r, v)$.
(b) For a minimum-spanning tree M: $\sum_{e \in E(T)} w(e) \le \beta \cdot \sum_{e \in E(M)} w(e)$.

(7 points)

Exercise 11.2. A posynomial function $f : \mathbb{R}^n_+ \to \mathbb{R}$ is of the form

$$f(x) = \sum_{k=1}^{K} c_k \prod_{i=1}^{n} x_i^{a_{ik}}$$

for $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$.

(a) Give an example for a non-convex posynomial function.

(b) Let f be a posynomial function with lower and upper bounds $l, u \in \mathbb{R}^n_+$, $l \le u$ on the variables. Show that each local minimum of f on the box $[l, u]$ is also a global minimum of f on $[l, u]$.

Hint: Use a logarithmic variable transformation to derive an equivalent convex problem.

(2 + 5 points)

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Exercise 11.3. Consider a chain of $n \in \mathbb{N}$ continuously sizable inverters with sizes $x_i > 0$ ($1 \leq i \leq n$) depicted in Figure 11.1. Assume that the delay

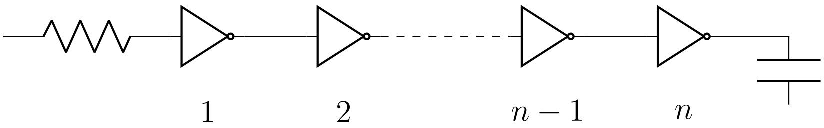

Figure 11.1: Chain of inverters.

θ_i through inverter i is given by

$$\theta_i(x) = \alpha + \frac{\beta \cdot x_{i+1}}{x_i} \qquad \text{for } 1 \leq i < n-1$$

where $x = (x_1, \ldots, x_n)$, $\alpha \geq 0$, $\beta > 0$. Wire delays, slews and transitions are ignored.

Derive a closed formula for the size x_i of the i-th inverter in a solution x of the total delay minimization problem for fixed x_1, x_n:

$$\min \left\{ \sum_{i=1}^{n-1} \theta_i(x) : x_i > 0 \text{ for all } 2 \leq i \leq n-1 \right\}.$$

(6 points)

Deadline: July 18th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss17/chipss17.html`

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.