Exercise Set 7

Exercise 7.1. Provide a polynomial time algorithm for the **Standard Placement Problem** restricted to instances with only one circuit.

(5 points)

Exercise 7.2. The **Gridded Placement Problem** is an extension of the **Standard Placement Problem** with a grid \( \Gamma = \Gamma_x \times \Gamma_y \) where \( \Gamma_z := \{ k \cdot \delta_z : k \in \mathbb{Z} \} \) with \( \delta_z \in \mathbb{Z} \) for \( z \in \{ x, y \} \). In this variant, the lower left corner of each circuit is required to be in \( \Gamma \).

Prove that the **Gridded Placement Problem** is NP-hard even if an optimum solution of the associated ungridded placement problem is known.

(5 points)

Exercise 7.3. Prove that unless P = NP, there is no polynomial time \( n^\alpha \) approximation algorithm for the **Quadratic Assignment Problem** for any \( \alpha < 1 \) even if \( w \equiv 1, c \equiv 0, d \) is metric and \( G \) is a tree.

(5 points)

Exercise 7.4. Let \( G = (V, E) \) be an undirected graph with edge weights \( w : E \rightarrow \mathbb{R}_{\geq 0} \) and \( k \in \mathbb{N} \). Let \( C \subseteq V \) and \( f : V \setminus C \rightarrow \{ 1, \ldots, k \} \) be a placement function. We are looking for positions \( f : C \rightarrow \{ 1, \ldots, k \} \) s.t.

\[
\sum_{e \in \{v,w\} \in E} w(e) \cdot |f(v) - f(w)|
\]

is minimum. Note that \( f \) is not required to be injective.

Prove that this problem can be solved optimally by solving \( k - 1 \) minimum weight \( s-t \)-cut problems in digraphs with \( \mathcal{O}(|V|) \) vertices and \( \mathcal{O}(|E|) \) edges.

**Hint:** Consider digraphs \( G_j = (V_j, E_j) \) with \( V_j := \{ s, t \} \cup C \) and

\[
E_j := \{(s,v) : \exists w \in V \setminus C, f(w) \leq j, \{v,w\} \in E\} \cup \\
\{(v,w) : v, w \in C, \{v,w\} \in E\} \cup \\
\{(v,t) : \exists w \in V \setminus C, f(w) > j, \{v,w\} \in E\}
\]

(5 points)
Deadline: June 20th, before the lecture. The websites for lecture and exercises can be found at

http://www.or.uni-bonn.de/lectures/ss17/chipss17.html

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.