Exercise Set 5

**Exercise 5.1.** Let $T \subset \mathbb{R}^2$ be a finite set of terminals, and $S_1, \ldots, S_m \subseteq \mathbb{R}^2$ be rectangular, axis-parallel blockages. Let $S := \bigcup_i S_i$, $\hat{S}$ denote the interior of $S$, and let $0 < L \in \mathbb{R}$ be a constant.

A rectilinear Steiner tree $Y$ for $T$ is **reach-aware** if every connected component of $E(Y) \cap \hat{S}$ has length at most $L$. We define the Hanan grid induced by $(T, S_1, \ldots, S_m)$ as the usual Hanan grid for $T \cup \{l_i, u_i \mid 1 \leq i \leq m\}$ where $l_i$ (resp. $u_i$) is the lower left (resp. upper right) corner of $S_i$.

Prove or disprove: There is always a shortest reach-aware Steiner tree for $T$ that is a subgraph of the Hanan grid induced by $(T, S_1, \ldots, S_m)$.

(4 points)

**Exercise 5.2.** Let $(G, c, T)$ be an instance of the Steiner Tree Problem, $G$ connected, $t \in T$ a terminal and $k \in \mathbb{N}$ with $k \geq 1$.

(a) For two valid lower bounds $lb_a$ and $lb_b$, define $\max(lb_a, lb_b)$ by

$$\max(lb_a, lb_b)(v, I) := \max \left( lb_a(v, I), lb_b(v, I) \right).$$

Show that $\max(lb_a, lb_b)$ also defines a valid lower bound.

(b) Prove that $lb_{BB}(v, I) := BB(\{v\} \cup I)$ is a valid lower bound for instances of the Rectilinear Steiner Tree Problem.

(c) Show that $lb_{mat}(v, I) := \frac{\text{mat}(\{v\})}{2}$ defines a valid lower bound.

(d) Define $lb_k(v, I) := \max \left\{ \text{mat}(J) \mid t \in J \subseteq I \cup \{v\}, |J| \leq k+1 \right\}$ if $t \in I$ and $lb_k(v, I) := 0$ otherwise. Show that $lb_k$ is a valid lower bound.

(3 + 1 + 1 + 4 points)
Exercise 5.3. Consider the following Clustered Rectilinear Steiner Tree Problem: Given a partition $T = \bigcup_{i=1}^{k} P_i$ of the terminals ($\emptyset \neq P_i \subseteq \mathbb{R}^2, |P_i| < \infty$), find a (rectilinear) Steiner tree $Y_i$ for each set of terminals $P_i$ and one rectilinear, toplevel (group) Steiner tree $Y_{\text{top}}$ connecting the embedded trees $Y_i$ ($i = 1, \ldots, k$). The task is to minimize the total length of all trees.

Let $A$ be an $\alpha$-approximation algorithm for the Rectilinear Steiner Tree Problem. A feasible solution to the Clustered Rectilinear Steiner Tree Problem can be found by first selecting a connection point $q_i \in \mathbb{R}^2$ for each $i = 1, \ldots, k$ and then computing $Y_i := A(P_i \cup \{q_i\})$ and $Y_{\text{top}} := A(\{q_i : 1 \leq i \leq n\})$.

(a) Show that picking $q_i \in P_i$ arbitrarily yields a $2\alpha$ approximation.

(b) Prove that choosing each $q_i$ as the center of the bounding box of $P_i$ implies a $\frac{7}{4}\alpha$ approximation algorithm.

(c) Show that both approximation ratios above are tight.

(2 + 4 + 2 points)

Deadline: May 30th, before the lecture. The websites for lecture and exercises can be found at

http://www.or.uni-bonn.de/lectures/ss17/chipss17.html

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.