Exercise 4.1. For a finite set $\emptyset \neq T \subseteq \mathbb{R}^2$ we define

$$BB(T) := \max_{(x,y) \in T} x - \min_{(x,y) \in T} x + \max_{(x,y) \in T} y - \min_{(x,y) \in T} y.$$ 

A *Steiner tree* for $T$ is a tree $Y$ with $T \subseteq V(Y) \subseteq \mathbb{R}^2$. We denote by $\text{Steiner}(T)$ the length of a shortest rectilinear (i.e. edge lengths acc. to $\ell_1$) Steiner tree for $T$. Moreover let $\text{MST}(T)$ be the length of a minimum spanning tree in the complete graph on $T$ with edge costs $\ell_1$.

Prove that:

(a) $BB(T) \leq \text{Steiner}(T) \leq \text{MST}(T)$;

(b) $\text{Steiner}(T) \leq \frac{3}{2} BB(T)$ for $|T| \leq 5$;

(c) There is no $\alpha \in \mathbb{R}$ s.t. $\text{Steiner}(T) \leq \alpha BB(T)$ for all finite $\emptyset \neq T \subset \mathbb{R}^2$.

(2 + 3 + 2 points)

Exercise 4.2. Let $T$ be an instance of the Rectilinear Steiner Tree Problem and $r \in T$. For a rectilinear Steiner tree $Y$ we denote by $f(Y)$ the maximum length of a path from $r$ to any element of $T \setminus \{r\}$ in $Y$.

(a) Find an instance where no Steiner tree minimizes both length and $f$.

(b) Consider the problem of finding a shortest Steiner tree $Y$ minimizing $f(Y)$ among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid?

(1 + 4 points)

Exercise 4.3. Let $Y$ be a Steiner tree for terminal set $T$ with $|T| \geq 2$ in which all leaves are terminals. Prove

$$\sum_{t \in T} \left( |\delta_Y(t)| - 1 \right) = k - 1$$

where $k$ is the number of full components of $Y$.

(5 points)
Deadline: May 18th, before the lecture. The websites for lecture and exercises can be found at

http://www.or.uni-bonn.de/lectures/ss17/chipss17.html

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de