Exercise Set 4

Exercise 4.1. For a finite set $\emptyset \neq T \subsetneq \mathbb{R}^2$ we define

$$\mathrm{BB}(T) := \max_{(x,y)\in T} x - \min_{(x,y)\in T} x + \max_{(x,y)\in T} y - \min_{(x,y)\in T} y.$$

A *Steiner tree* for T is a tree Y with $T \subseteq V(Y) \subsetneq \mathbb{R}^2$. We denote by Steiner($T$) the length of a shortest rectilinear (i.e. edge lengths acc. to ℓ_1) Steiner tree for T. Moreover let MST(T) be the length of a minimum spanning tree in the complete graph on T with edge costs ℓ_1.

Prove that:

(a) $\mathrm{BB}(T) \leq \mathrm{Steiner}(T) \leq \mathrm{MST}(T)$;

(b) $\mathrm{Steiner}(T) \leq \frac{3}{2}\,\mathrm{BB}(T)$ for $|T| \leq 5$;

(c) There is no $\alpha \in \mathbb{R}$ s.t. $\mathrm{Steiner}(T) \leq \alpha\,\mathrm{BB}(T)$ for all finite $\emptyset \neq T \subset \mathbb{R}^2$.

(2 + 3 + 2 points)

Exercise 4.2. Let T be an instance of the Rectilinear Steiner Tree Problem and $r \in T$. For a rectilinear Steiner tree Y we denote by $f(Y)$ the maximum length of a path from r to any element of $T \setminus \{r\}$ in Y.

(a) Find an instance where no Steiner tree minimizes both length and f.

(b) Consider the problem of finding a shortest Steiner tree Y minimizing $f(Y)$ among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid?

(1 + 4 points)

Exercise 4.3. Let Y be a Steiner tree for terminal set T with $|T| \geq 2$ in which all leaves are terminals. Prove

$$\sum_{t\in T} \Big(|\delta_Y(t)| - 1\Big) = k - 1$$

where k is the number of full components of Y.

(5 points)

Deadline: May 18th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss17/chipss17.html`

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.