Research Institute for Discrete Mathematics
Approximation Algorithms
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Repetition Sheet

Exercise 1: $\mathcal{O}$, Θ, Ω-notation
Determine if $f(n) \in \mathcal{O}(g(n))$, $f(n) \in \Omega(g(n))$, $f(n) \in \Theta(g(n))$

1. $f(n) = n^3$, $g(n) = 3n^3 + n^2 + 7n - 100$
2. $f(n) = n^{\log(n)}$, $g(n) = n^2$
3. for $\epsilon > 0$: $f(n) = \log(n)$, $g(n) = n^\epsilon$
4. $f(n) = \log(n!)$, $g(n) = n\log(n)$
5. $f(n) = \log(1 + \frac{1}{n})$, $g(n) = \frac{1}{n}$

Exercise 2: Turing machines - The Halting Problem
Turing machines can be encoded by binary strings. The Halting Problem is defined as follows: Given two binary strings x and y, with x encoding a Turing machine Φ. Question: Does Φ with input y terminate after finite time?
Prove that the Halting Problem is undecidable, i.e. there exists no algorithm which solves this problem.

Exercise 3: NP
Prove: If $\mathcal{P} \in$ NP, there exists a polynomial p such that $\mathcal{P}$ can be solved by a deterministic algorithm in time $\mathcal{O}(2^{p(n)})$.

Exercise 4: Graph theory
Prove that HAMILTONIAN PATH and HAMILTONIAN CYCLE are not NP-complete for simple, undirected graphs G with $|\delta(v)| \geq \frac{n}{2}$ for each $v \in V(G)$ $(n := |V(G)|)$.

Exercise 5: NP hardness
Prove that the following problem is NP-complete:

Instance: An instance of 3SAT.
Question: Is there a truth assignment in which every clause contains at least one true and one false literal?