Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

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# Exercise Set 11

**Exercise 11.1:**
Given a complete graph $G$, $s, t \in V(G)$ and weights $c : E(G) \to \mathbb{R}_+$ satisfying the triangle inequality, consider the problem to find an $s$-$t$ path containing all vertices of $G$ of minimum total weight.
Generalize Christofides' Algorithm to obtain a $\frac{5}{3}$-approximation.

(4 points)

**Exercise 11.2:**
Let $G$ be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for the Traveling Salesman Problem in this special case.
*Hint: You may use that a minimum 2-matching in $G$, i.e. a subgraph of $G$ in which every vertex has degree 2, can be computed in polynomial time.*

(4 points)

**Exercise 11.3:**
Describe a polynomial-time algorithm which optimally solves any instance of the Traveling Salesman Problem that is the metric closure of a weighted tree.

(4 points)

**Exercise 11.4:**
The Metric Bipartite Traveling Salesman Problem is the problem of finding a Hamiltonian circuit of minimum cost in a bipartite graph $H$ with a nonnegative cost function $d$ satisfying

$$d(\{a,b\}) + d(\{a',b\}) + d\{a',b'\}) \geq d(\{a,b'\}) \text{ for } \{a,b\}, \{a',b\}, \{a,b'\}, \{a',b'\} \in E(H).$$

Prove that for any $k$, if there is a $k$-factor approximation algorithm for the Metric Bipartite Traveling Salesman Problem, there is also a $k$-factor approximation algorithm for the Metric Traveling Salesman Problem.

*Hint: Given an instance $(G, c)$ of the* Metric TSP, *construct an instance $(H, d)$ of the* Metric Bipartite TSP *where $V(H) := V(G) \times \{0, 1\}$ and $d\big(\{(v,0),(w,1)\}\big) \in \big\{c(\{v,w\}), 0\big\}$.*

(4 points)

**Deadline:** Thursday, July 13th, before the lecture.