Exercise Set 11

Exercise 11.1:
Given a complete graph $G$, $s,t \in V(G)$ and weights $c : E(G) \to \mathbb{R}_+$ satisfying the triangle inequality, consider the problem to find an $s$-$t$ path containing all vertices of $G$ of minimum total weight.
Generalize Christofides’ Algorithm to obtain a $\frac{5}{3}$-approximation.

(4 points)

Exercise 11.2:
Let $G$ be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{3}{2}$-approximation algorithm for the Traveling Salesman Problem in this special case.

Hint: You may use that a minimum 2-matching in $G$, i.e. a subgraph of $G$ in which every vertex has degree 2, can be computed in polynomial time.

(4 points)

Exercise 11.3:
Describe a polynomial-time algorithm which optimally solves any instance of the Traveling Salesman Problem that is the metric closure of a weighted tree.

(4 points)

Exercise 11.4:
The Metric Bipartite Traveling Salesman Problem is the problem of finding a Hamiltonian circuit of minimum cost in a bipartite graph $H$ with a nonnegative cost function $d$ satisfying

$$d(\{a, b\}) + d(\{a', b\}) + d(\{a', b'\}) \geq d(\{a, b'\}) \text{ for } \{a, b\}, \{a', b\}, \{a, b'\}, \{a', b'\} \in E(H).$$

Prove that for any $k$, if there is a $k$-factor approximation algorithm for the Metric Bipartite Traveling Salesman Problem, there is also a $k$-factor approximation algorithm for the Metric Traveling Salesman Problem.

Hint: Given an instance $(G, c)$ of the Metric TSP, construct an instance $(H, d)$ of the Metric Bipartite TSP where $V(H) := V(G) \times \{0, 1\}$ and $d(\{(v, 0), (w, 1)\}) \in \{c(\{v, w\}), 0\}$.

(4 points)

Deadline: Thursday, July 13th, before the lecture.