Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

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Exercise Set 10

Exercise 10.1:
In this exercise we consider variants of the contraction lemma.

(i) Show that the "vertex version" of the contraction lemma is wrong: Define a complete graph whose edge lengths fulfill the triangle inequality and vertex sets A, B, and C such that

$$0 < \mathrm{mst}(A) - \mathrm{mst}(A \cup C) < \mathrm{mst}(A \cup B) - \mathrm{mst}(A \cup B \cup C).$$

Here, $\mathrm{mst}(X)$ for a vertex set X denotes the length of a minimum spanning tree in the graph induced by X.

(ii) Show that the contraction lemma still holds if one adds edges between terminals whose lengths are larger than 0. ("Adding" an edge means that if there already is an edge, a parallel edge is inserted.)

(4+4 points)

Exercise 10.2:
Consider an instance $G = (V, E)$ of the Steiner Tree Problem with terminal set R and edge lengths $c\colon E \to \mathbb{R}_+$. Denote the full components of an optimum k-Steiner tree $\mathrm{SMT}_k(R)$ with $T_1^*, \ldots, T_k^*$.

(i) Suppose that $V \setminus R$ forms a stable set. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_k^*))\,.$$

(ii) Suppose that all shortest paths between any two vertices in G have length 1 or 2. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_k^*))\,.$$

(iii) Show that the Loss Contraction Algorithm achieves an approximation ratio of $1.279 \cdot r_k$ in both cases.

(2+2+2 points)

Exercise 10.3:
Show that the approximation ratio of the Relative Greedy algorithm is not better than $\frac{41}{30}$. *Hint:* *turn page*

(4 points)

Deadline: Thursday, July 6th, before the lecture.

Hint for Exercise 10.3:

Construct instances G_k with

- $4k$ terminals $w_1, \ldots, w_{4k}$,
- three special vertices T_a, T_b, T_c,
- $2k$ vertices $y_1, \ldots, y_{2k}$,
- k vertices $z_1, \ldots, z_k$.

Insert edges as follows:

- each terminal w_{4i-3} is connected to T_a by an edge of length x, where $2 < x < 2.5$
- each terminal w_{4i} and w_1 is connected to T_b by an edge of length x
- each y_i is connected to w_{2i-1} and w_{2i} by an edge of length 1
- each z_i is connected to y_{2i-1} and y_{2i} by an edge of length $\frac{1}{2}$
- each z_i is connected to T_c by an edge of length 1
- each terminal w_{4i-3} is connected to w_{4i-2} by an edge of length 2
- each terminal w_{4i-2} is connected to w_{4i-1} by an edge of length 3
- each terminal w_{4i-1} is connected to w_{4i} by an edge of length 2
- each terminal w_{4i} is connected to w_{4i+1} by an edge of length $2x$.

The graph G_4 looks as follows: