Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

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# Exercise Set 9

**Exercise 9.1:**
Show that in Mehlhorn's algorithm replacing the edges of the minimum spanning tree by corresponding shortest paths does not result in cycles.
*Note: You may use that the Voronoi regions are computed with Dijkstra's algorithm.*
(4 points)

**Exercise 9.2:**
Consider the following greedy algorithm for the GRAPH STEINER TREE PROBLEM: Given a graph $G = (V, E)$ with terminal set $R$ and edge lengths $c\colon E \to \mathbb{R}_+$ we compute a minimum spanning tree $T = \mathrm{MST}(R)$ in the terminal distance graph $G_D(R)$. While there is some $v \in V(G)\setminus R$ with $c(\mathrm{MST}(R \cup \{v\})) < c(\mathrm{MST}(R))$ set $R := R \cup \{v\}$ and remove any non-terminals of degree $\leq 2$ (in $\mathrm{MST}(R)$) from $R$. Return $\mathrm{MST}(R)$.
Suppose that $V\setminus R$ forms a stable set.

(i) Show that this algorithm is a $\frac{3}{2}$-approximation algorithm. (4 points)

(ii) Show that this algorithm is no $\rho$-approximation for any $\rho < \frac{3}{2}$. (4 points)

**Exercise 9.3:**
Let $R \subseteq \mathbb{R}^2$ be a finite set containing a vertex $s \in R$. Let $G$ be the complete graph with vertex set $\{(p_x, q_y) \mid p, q \in R\}$, where we denote by $p_x$ and $p_y$ the $x$- and $y$-coordinate of $p \in \mathbb{R}^2$ respectively. The length of an edge is defined as the $L_1$-distance between its endpoints (i.e. $\mathrm{length}(\{p, q\}) := |p_x - q_x| + |p_y - q_y|$).
We want to compute a Steiner tree $T$ for $R$ in $G$ such that the length of the unique $s$-$r$ path in $T$ is shortest possible for each $r \in R$.

(i) Give a polynomial time algorithm that computes such a Steiner tree $T$ with

$$\sum_{\{p,q\}\in E(T)} \mathrm{length}(p, q) \leq \lceil \log(|R| - 1) \rceil \cdot \mathrm{mst}(R).$$

(ii) Show that the bound of (i) is best possible up to a constant factor.

(4+2 points)

**Deadline:** Thursday, June 29th, before the lecture.