Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

Prof. Dr. S. Hougardy
D. Rotter

Exercise Set 8

Exercise 8.1:
Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum-cost subgraph of G that contains a path connecting each receiver to a sender.

(i) Prove that the restriction of this problem to instances with $S \cup R = V$ is in P.

(ii) Prove that the restriction of this problem to instances with $S \cup R \subsetneq V$ is NP-hard and give a 2-factor approximation algorithm.

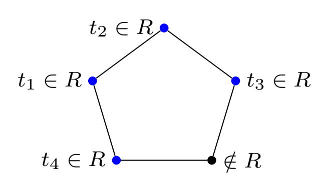
(3+3 points)

Exercise 8.2:
Let T be a Steiner tree for a terminal set R. We may assume that T is a binary tree and its leaves are exactly R. We would like to turn $r \leq |V(T)\backslash R|$ Steiner nodes into terminals such that we obtain a k'-Steiner tree with k' minimum. Give a polynomial time algorithm that solves this problem optimally.

(4 points)

Exercise 8.3:
Let $G = (V, E)$ be an undirected planar graph and let $R \subseteq V$ be a set of terminals that are all lying on the outer face.

A set $U \subseteq R$ is called *consecutive* if there is a path P whose vertices all lie on the outer face and $V(P) \cap R = U$.
In the graph depicted on the right, $\{t_1, t_2\}$ is consecutive but $\{t_1, t_3\}$ is not.



(i) Show that it is enough to consider consecutive sets in Lines 3 and 5 of the Dreyfus-Wagner algorithm.

(ii) Conclude that a shortest Steiner tree for G can be computed in $\mathcal{O}(|V|^3|R|^2)$ time.

(4+2 points)

Deadline: Thursday, June 22th, before the lecture.

Note: There is no lecture on June 15th. You can hand in the 7th sheet tomorrow in the exercise classes, before the lecture on June 20th, or in my office until June 20th.