Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

Prof. Dr. S. Hougardy
D. Rotter

Exercise Set 7

Exercise 7.1:
In this exercise we will derive a polynomial algorithm for a variant of the Bin Packing problem where we relax the bin capacities by some $\epsilon > 0$.

(i) Give an algorithm for Bin Packing restricted to instances with a constant number of different item sizes whose running time is polynomially bounded in the number n of items.

Hint: Use dynamic programming.

(ii) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which for any instance $I = (a_1, \dots, a_n)$ of the Bin Packing problem finds a packing using the optimum number of bins but possibly violating the capacity constraints by ε, i. e. an $f : \{1, \dots, m\} \to \{1, \dots, \mathrm{OPT}(I)\}$ with $\sum_{f(i)=j} a_i \leq 1 + \varepsilon$ for all $j \in \{1, \dots, \mathrm{OPT}(I)\}$.

(4+4 points)

Exercise 7.2:
Consider the following algorithm for the Graph Steiner Tree Problem with 3 terminals v_1, v_2, and v_3: Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z minimizing $\sum_{i=1}^{3} \mathrm{dist}(v_i, z)$ under the conditions

(i) $\mathrm{dist}(v_i, z) \leq \mathrm{dist}(v_1, v_2)$ for $i \in \{1, 2\}$ and

(ii) $\mathrm{dist}(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm needs $\mathcal{O}(|E| + |V| \log |V|)$ time and works correctly.

(4 points)

Deadline: Tuesday, June 20th, before the lecture.