Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

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# Exercise Set 5

**Exercise 5.1:**
Consider the linear-time WEIGHTED MEDIAN algorithm. Recall that the list of numbers is partitioned into groups of 5 elements each. Does the algorithm still have linear runtime if the numbers are instead partitioned into

(i) groups of 3 elements each? (2 points)

(ii) groups of 7 elements each? (2 points)

**Exercise 5.2:**
Consider the following generalization of the KNAPSACK problem:

**Instance:** An instance $w_i, \dots, w_n, c_1, \dots, c_n, W$ of the KNAPSACK problem, values $b_1, \dots b_n \in \mathbb{Z}_+ \cup \{\infty\}$.
**Output:** Integers $0 \le x_i \le b_i$ for $i = 1, \dots, n$ such that $\sum_{i=1}^n x_i w_i \le W$ and $\sum_{i=1}^n x_i c_i$ is maximum.

(i) Prove that this problem is NP-hard even if $b_i = \infty$ for $i = 1, \dots, n$. (4 points)

(ii) Give an FPTAS for this problem. (4 points)

**Exercise 5.3:**
Describe a polynomial-time combinatorial algorithm for the FRACTIONAL MULTI KNAPSACK PROBLEM:

**Instance:** Natural numbers $n, m \in \mathbb{N}$ and $w_i \in \mathbb{N}$, $W_j \in \mathbb{N}$, and $c_{ij} \in \mathbb{N}$ for $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m\}$.
**Output:** Values $x_{ij} \in \mathbb{R}_{\ge 0}$ for $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m\}$ such that

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad \text{for all } i \in \{1, \dots, n\},$$

$$\sum_{i=1}^{n} x_{ij} w_i \le W_j \qquad \text{for all } j \in \{1, \dots, m\},$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} c_{ij} \qquad \text{is minimum.}$$

(4 points)

**Deadline:** Thursday, May 25th, before the lecture.