Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

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Exercise Set 4

Exercise 4.1:
Describe an algorithm which decides if an undirected graph $G = (V, E)$ is 4-colorable in time $\mathcal{O}(|E| \cdot 2^{|V|})$.

(4 points)

Exercise 4.2:
The LP relaxation of the MINIMUM WEIGHT VERTEX COVER PROBLEM is

$$\min\{cx : M^T x \geq 1, x \geq 0\},$$

where $M \in \{0,1\}^{n\times m}$ is the incidence matrix of an undirected graph G and $c \in \mathbb{R}^n_+$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

(i) Show that the above LP relaxation has a half-integral optimum solution.

(ii) Given a graph $G = (V, E)$ with weights $c \in \mathbb{R}^n_+$ and a coloring $\varphi : V \to \{1, \ldots, k\}$. Show how the LP relaxation can be used to find a vertex cover $X \subseteq V$ with cost $c(X) \leq \left(2 - \frac{2}{k}\right) \cdot \mathrm{OPT}$. Here, OPT denotes the cost of an optimum solution.

(3+3 points)

Exercise 4.3:

Let G be a k-colorable graph with n vertices, where k is a constant. We define $x_k := n^{1-\frac{1}{k-1}}$ and for $2 \leq l < k$, $x_l := {x_{l+1}}^{1-\frac{1}{l-1}}$. For simplicity, we assume that n is chosen such that x_l is a natural number for $l \in \{2, \ldots, k\}$.

Prove that there exists a polynomial time algorithm that colors G with kx_k colors.

(6 points)

Deadline: Thursday, May 18th, before the lecture.