Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

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Exercise Set 3

Exercise 3.1:
Consider the following variant of the k-center problem:

Instance: A complete graph $G = (V, E)$, metric edge weigths $d : E(G) \to \mathbb{R}$, a partition $V = C \dot{\cup} S$, an integer $k \in \mathbb{N}$.
Output: A set $X \subseteq S$ with $|X| \le k$ such that

$$\max_{c \in C} \left\{ \min_{s \in X} \{d(c, s)\} \right\} \quad \text{is minimum.}$$

(i) Show that this problem does not admit a $(3 - \epsilon)$-approximation for any $\epsilon > 0$ unless P=NP. (4 points)

(ii) Give a 3-approximation algorithm. (4 points)

Exercise 3.2:
Consider the following variant of SET COVER:

Instance: A set U, sets $\mathcal{S} = \{S_1, \ldots, S_m\}$ such that $\bigcup_{i=1}^{m} S_i = U$, an integer $k \in \mathbb{N}$.
Output: k sets $S_{i_1}, \ldots, S_{i_k} \in \mathcal{S}$ such that $\left| \bigcup_{j=1,\ldots,k} S_{i_j} \right|$ is maximum.

Show that iteratively picking the element that maximizes the amount of not yet covered elements is a $\left(1 - \frac{1}{e}\right)$-approximation.

(4 points)

Exercise 3.3:
An instance of MAXIMUM SATISFIABILITY is called k-satisfiable if any k of its clauses can be satisfied simultaneously. Let r_k be the infimum of the fraction of clauses that one can satisfy in any k-satisfiable instance.

Prove that $r_2 = \frac{\sqrt{5}-1}{2}$ and give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a r_2-fraction of the clauses.

(4 points)

Deadline: Thursday, May 11th, before the lecture.