Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

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Exercise Set 2

Exercise 2.1:
For $k \in \mathbb{N}$ consider the following problem:

Instance: A set U and a set $\mathcal{S}$ of subsets of U with $|S| \leq k$ for all $S \in \mathcal{S}$, weights $w : U \to \mathbb{R}_{\geq 0}$.
Task: Find $T \subseteq U$ such that $T \cap S \neq \emptyset$ for each $S \in \mathcal{S}$ and $\sum_{t \in T} w(t)$ minimum.

(i) Show that this problem is NP-hard for $k \geq 2$. (2 points)

(ii) Give a polynomial time k-factor approximation algorithm. (3 points)

(iii) Give a *linear time* k-factor approximation algorithm for the case that $w(t) = 1$ for $t \in U$. (3 points)

Exercise 2.2:
Prove that the following problem is NP-complete:

Instance: An undirected graph $G = (V, E)$ and an integer k.
Question: Is there $X \subset V$ with $|X| \leq k$ and $|\delta(X)| \geq k$?

(4 points)

Exercise 2.3:
Given a directed cycle $C = (V, E)$ and a set of undirected edges $E_1 \subseteq \{\{v, w\} | v, w \in V, v \neq w\}$. We are looking for an orientation $E_1^{\leftrightarrow}$ of E_1 such that, in the digraph $G' = (V, E \cup E_1^{\leftrightarrow})$,

$$\max_{e \in E} \left| \{C' \text{ directed cycle} \mid e \in E(C') \text{ with } |E(C') \cap E_1^{\leftrightarrow}| = 1\} \right|$$

is minimum. Give a linear time 2-approximation algorithm for that problem.
(4 points)

Deadline: Thursday, May 4th, before the lecture.