Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2017

Prof. Dr. S. Hougardy
D. Rotter

Exercise Set 1

Exercise 1.1:
Prove that SATISFIABILITY remains NP-complete if each clause contains at most three literals and each variable occurs in at most three clauses.

(4 points)

Exercise 1.2:
Show that the following problem is NP-complete:

Instance: A directed graph G.
Question: Is there some $X \subset V$ such that $E(G[X]) = \emptyset$ and that for all $v \in V \backslash X$ we have $\delta^+_{G[X \cup \{v\}]}(v) \neq \emptyset$?

Hint: Use a reduction from SATISFIABILITY.

(4 points)

Exercise 1.3:
Show that the following problem is not NP-complete for every $k \geq 1$ unless P=NP:

Instance: A k vertex connected undirected graph G that does not contain a stable set with $k+1$ vertices.
Question: Does G contain a hamiltonian cycle?

(4 points)

Definition.
For $\tau \leq 1$, a *τ-approximation algorithm* for the maximum stable set problem is a polynomial time algorithm that computes for every undirected graph $G = (V, E)$ a stable set $S \subset V$ such that $|S| \geq \tau \cdot \max\{|S^*| \mid S^* \subset V \text{ is a stable set}\}$.

Exercise 1.4:
Prove: If there is a $\frac{1}{2}$-approximation algorithm for the maximum stable set problem, there is also a $(1-\epsilon)$-approximation algorithm for every $\epsilon > 0$.

(4 points)

Deadline: Thursday, April 27th, before the lecture.

The websites for lecture and exercises are linked at

http://www.dm.uni-bonn.de/lectures/ss17/ss17.html

In case of any questions feel free to contact me at rotter@dm.uni-bonn.de or 0228 73 8750.