CR-Covering

Sunday, May 29, 2016 9:32 PM

A general 0,1-covering prob. (0,1-CIP) is of the form

$$\begin{aligned} \min \quad & c^T x & A \in \{0,1\}^{m \times n} \\ \text{s.t.} \quad & Ax \geq \mathbb{1} & \\ & x \in \{0,1\}^m & \end{aligned}$$

well understood in terms of approximability

[Chvatal '79] $O(\log n)$ greedy alg

[Feige '98] no $c \cdot \log(n)$-apx for $c < 1$ exists unless $NP \subseteq DTIME(n^{O(\log \log n)})$

<u>Positive results</u>

① Column-sparse instances

holds for general CIPs ($A \geq 0$)

If A has $\leq \alpha$ positive entries in each column

$\Longrightarrow \exists\ O(1 + \log \alpha)$-apx

[Srinivasan '99], [Kolliopoulos, Young '05]

② Row-sparse instances

If A has $\leq \beta$ positive entries per row

$\Longrightarrow \beta$-apx

[Pritchard & Chakrabarty '09]

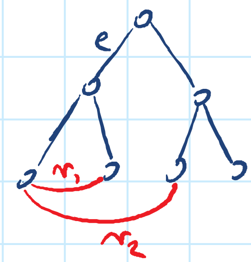
Here: study natural capacitated version

Ex: Tree augmentation

Given tree $T = (V, E)$ and <u>links</u> $\mathcal{L} \subseteq V \times V$

Find min card $F \subseteq \mathcal{L}$
s.t. $T + F = (V, E + F)$
is 2-edge connected

$\forall e \in E$, let $\mathcal{L}_e \subseteq \mathcal{L}$ de those links whose fund. cycle contains e.



$r_2 \in \mathcal{L}_e$, $r_1 \notin \mathcal{L}_e$

(IP)
$$\begin{aligned} \min\ & \mathbb{1}^T x \\ \text{s.t. } & x(\mathcal{L}_e) \geq 1 \quad \forall e \in E \\ & x \geq 0 \\ & x \text{ int.} \end{aligned}$$

Capacitated version each $\ell \in \mathcal{L}$ has a supply $s_\ell \geq 0$
and each edge $e \in E$ a demand $b_e \geq 0$

goal: find $F \subseteq \mathcal{L}$ of smallest card s.t.

$$s(\mathcal{L}_e) \geq b_e \quad \forall e \in E$$

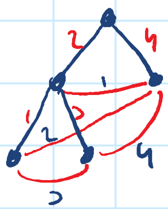
Model (a) starting point 0,1-CIP

0,1 CIP $\quad \min\{c^T x : Ax \geq \mathbb{1},\ x \geq 0,\ x \text{ int}\}$

ⓑ Add supply $s_j \geq 0$, demand $b_i \geq 0$ $\forall i,j$

ⓒ Replace A_{ij} by $A_{ij} \cdot s_j$ $\forall i,j$

$\Longrightarrow A[s]$



$$\underbrace{\begin{aligned} &\min \ \mathbb{1}^T x \\ &\text{s.t.} \ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}_A x \geq \mathbb{1} \\ &x \geq 0 \\ &x \text{ int} \end{aligned}}_{\text{underlying 0,1-CIP}} \Longrightarrow \underbrace{\begin{aligned} &\min \ \mathbb{1}^T x \\ &\text{s.t.} \ A[s] \begin{pmatrix} 0 & s_2 & s_3 & 0 \\ s_1 & s_2 & 0 & s_4 \\ 0 & 0 & s_3 & s_4 \\ s_1 & s_2 & 0 & s_4 \end{pmatrix} x \geq b \\ &x \geq 0 \\ &x \text{ int} \end{aligned}}_{\text{CRIP}}$$

Q: How well can we approx.

(CCIP) $\quad \min \ c^T x$

s.t. $\quad A[s]\, x \geq b$

$\quad 0 \leq x \leq d$

Well: in general not much better than $O(\log m)$... but what if the underlying 0,1-CIP is nicely structured?

Priority-Covering IP $\quad$ Given 0,1-CIP and priorities s_j, π_i $\forall i,j$

Say column j covers row i if

(a) $A_{ij} = 1$, and
(b) $s_j \geq \pi_i$

$$\Rightarrow \quad A[s,\pi]_{ij} = \begin{cases} 1 & : A_{ij}=1 \wedge s_j \geq \pi_i \\ 0 & : \text{othw.} \end{cases}$$

Priority-Covering IP

(PCIP) $\quad \min \{ c^T x : A[s,\pi]\, x \geq \mathbb{1},\ 0 \leq x \leq d,\ x \text{ int} \}$

Two assumptions

Given an instance of CCIP

(A1) Integrality-gap of underlying 0,1-CIP is $\leq \alpha$

$\Rightarrow \forall b, c, d \geq 0$ if x feasible for

$$\min \{ c^T x : A x \geq b,\ 0 \leq x \leq d \}$$

then exists integral feas. soln $\bar{x}$ of cost $\leq \alpha\, c^T x$.

(A2) Integrality gap of PCIP is $\leq \omega$

$\Rightarrow \forall s, \pi, c \geq 0$ if x feas for

$$\min \{ c^T x : A[s,\pi]\, x \geq \mathbb{1},\ x \geq 0 \}$$

then exists int. feas soln $\bar{x}$ of cost $\leq \omega\, c^T x$

Thm [Chakrabarty, Grant, K. '10]

(A1), (A2) $\Longrightarrow$ $(24\gamma + 8\omega)$-apx for CCIP