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Exercise Set 9

Exercise 9.1:
Prove Kraft's inequality (Proposition 4.1 in the lecture notes).
Let S be a nonempty finite set, $r \notin S$, and $h_s \in \mathbb{N}$ for $s \in S$. There exists a topology for root r and sinks S with $\left|E\left(A_{[r,s]}\right)\right| - 1 \leq h_s$ for all $s \in S$ if and only if

$$\sum_{s \in S} 2^{-h_s} \leq 1.$$

(5 points)

Exercise 9.2:
Consider the Repeater Tree Topology problem. Let $|S| = n \geq 2$, and for each sink s_i define $a'_{s_i} := a_{s_i} - d\|p(r) - p(s_i)\|_1$, $i = 1, \ldots, n$.

Algorithm 1: Huffman coding

$A \leftarrow (\{r\}, \emptyset)$;
for $k = n, n-1, \ldots, 2$ **do**
  Sort $S = \{s_1, \ldots, s_k\}$ s.t. $a'_{s_1} \leq \cdots \leq a'_{s_k}$;
  Merge two sinks s_{k-1}, s_k into a new sink s'_{k-1};
  Choose $p(s'_{k-1})$ as the position that minimizes

$$\|p(r) - p(s'_{k-1})\|_1 + \|p(s_k) - p(s'_{k-1})\|_1 + \|p(s_{k-1}) - p(s'_{k-1})\|_1;$$

  Set $a_{s'_{k-1}} := \min\{a_{s_i} - d\,\|p(s_i) - p(s'_{k-1})\|_1 - b \mid i = k, k-1\}$;
  `/* Update` S`, add vertices and edges to` A `*/`
  $S \leftarrow (S \setminus \{s_{k-1}, s_k\}) \cup \{s'_{k-1}\}$;
  $V(A) \leftarrow V(A) \cup \{s_{k-1}, s_k, s'_{k-1}\}$;
  $E(A) \leftarrow E(A) \cup \{(s'_{k-1}, s_{k-1}), (s'_{k-1}, s_k)\}$;
`/* Connect the last vertex to` r `*/`
$E(A) \leftarrow E(A) \cup \{(r, s'_1)\}$.

Show that the topology generated by Algorithm 1 maximizes the worst slack.
(5 points)

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Exercise 9.3:
Show that the approximation algorithm for the Rectilinear Sink Clustering Problem presented in the lecture can be implemented to run in $O(|D| \log |D|)$ time.

Note: you can assume that a shortest rectilinear spanning tree on n terminals can be computed in $O(n \log n)$ time.

(5 points)

Deadline: June 30th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss16/ss16.html`

In case of any questions feel free to contact me at saccardi@or.uni-bonn.de.