Exercise Set 8

Exercise 8.1:
Consider the Placement Legalization Problem with $y_{\max} - y_{\min} = 1$. We are given an infeasible placement $\tilde{x} : \mathcal{C} \to \mathbb{R}$. Show that there are instances for which there is no optimum solution which is consistent with $\tilde{x}$, i.e. such that $x(C) < x(C') \Rightarrow \tilde{x}(C) \leq \tilde{x}(C')$.
(4 points)

Exercise 8.2:
Consider the following variant of the Single Row Placement With Fixed Ordering problem, in which we minimize the bounding box net length:

Input: A set $\mathcal{C} = \{C_1, \ldots, C_n\}$ of circuits, widths $w(C_i) \in \mathbb{R}_+$, an interval $[0, w(\square)]$, s.t. $\sum_{i=1}^n w(C_i) \leq w(\square)$. A netlist $(\mathcal{C}, P, \gamma, \mathcal{N})$ where the offset of a pin $p \in P$ satisfies $x(p) \in [0, w(\gamma(p))]$. Weights $\alpha : \mathcal{N} \to \mathbb{R}_+$.

Task: Find a feasible placement given by a function $x : \mathcal{C} \to \mathbb{R}$ s.t. $0 \leq x(C_1)$, $x(C_i) + w(C_i) \leq x(C_{i+1})$ for $i = 1, \ldots, n-1$ and $x(C_n) + w(C_n) \leq w(\square)$, that minimizes
$$\sum_{N \in \mathcal{N}} \alpha(N) \cdot \mathrm{BB}(N).$$

Here, $\mathrm{BB}(N)$ denotes the bounding box net length.
Show that there exist $f_i : [0, w(\square)] \to \mathbb{R}$, $i = 1, \ldots, n$, piecewise linear, continuous and convex, such that we can solve this problem by means of the Single Row Algorithm.
(4 points)

Exercise 8.3:
Give a detailed proof of Corollary 3.27, i.e. show that the variant of the Placement Legalization Problem in which the goal is to minimize $\sum_{C \in \mathcal{C}} |x(C) - \tilde{x}(C)|$ is strongly NP-hard even if $\mathcal{S} = \emptyset$ and $y_{\max} - y_{\min} = 1$.
(3 points)

Exercise 8.4: **Programming exercise**
Implement the dynamic programming buffering algorithm proposed by van Ginneken, with runtime $O\left(|L|^2 |V(A)|^2\right)$. Your program will take one argument, the path to a file which contains the input, and print the result to the standard output. The task is to find a buffering that maximizes the worst slack. *Buffers can be placed only at vertices v with $|\delta^+(v)| = |\delta^-(v)| = 1$.*

Input: the first line of the input contains the number of vertices $n = |V(A)|$. The set of vertices is $\{0, \ldots, n-1\}$. Each of the following $n-1$ lines contains four natural numbers $v\ w\ C_e\ R_e$, encoding an edge $e = (v, w)$ with capacitance C_e and resistance R_e. You may assume that this graph is an arborescence rooted at 0, in which every leaf is a sink.

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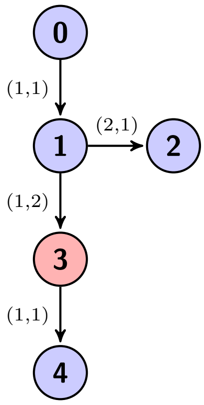
```
5
0 1 1 1
1 2 2 1
1 3 1 2
3 4 1 1
1
2 1
```

(a) Input file, with a library consisting of one single buffer.



(b) Represented tree. The values on the edges are (C_e, R_e).

```
-26 1
3 1
```

(c) Example output.

Figure 1: Example input and output, where a buffer of type 1 is placed at vertex 1. Buffers can be placed only at red vertices.

The next line contains the number of buffer types $|L|$ in the library. Each of the following lines contains two natural numbers C_l R_l, where C_l is the input capacitance and R_l the resistance of a buffer $l \in L$. The delay of a circuit (buffer or circuit at the root) with resistance R is $R \cdot \mathrm{dc}$, where dc is the downstream capacitance.
For every sink v, assume $\mathrm{rat}(v) = 0$, and input capacitance $C_v = 1$. Assume that the root has resistance $R_0 = 1$.

Output: the output must start with a line containing the worst slack and the number of buffers b. This should then be followed by b lines, consisting of two integers v l, which means that a buffer of type l is used at vertex v.

The program must be written in C or C++ (you are allowed to use up to C++11) and must compile and run on Linux. To achieve the maximum score, your program must not leak any memory and must be well documented. It must compile with either Clang $\geq$ 3.4.2 or Gcc $\geq$ 4.8.3, with `-Wall -Wpedantic -Werror`, and it cannot link to any other library. You can use any tool available in the standard library.

The deadline for the programming exercise is July 3rd, 12:00. Deliver your source code by email.

(20 points)

Deadline: June 23rd, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss16/ss16.html`

In case of any questions feel free to contact me at saccardi@or.uni-bonn.de.