Exercise Set 7

Exercise 7.1:
Consider Lemma 3.13 and the spreading LP (3.10) for $d = 2$:

$$
\begin{array}{llr}
\min & \displaystyle\sum_{e \in E(G)} w(e)\, l(e) & \\
\text{s.t.} & \displaystyle\sum_{y \in X} l(\{x,y\}) \geq \frac{1}{4}\left(|X|-1\right)^{3/2} & x \in X \subseteq V(G) \\
& l(\{x,y\}) + l(\{y,z\}) \geq l(\{x,z\}) & x,y,z \in V(G) \\
& l(\{x,y\}) \geq 1 & x,y \in V(G),\ x \neq y \\
& l(\{x,x\}) = 0 & x \in V(G)
\end{array}
$$

Let L be the optimum value of the spreading LP. Show that L is a lower bound for the cost of any 2-dimensional arrangement (i.e. show that for $d = 2$, one can choose $c_d = 1$).

(5 points)

Exercise 7.2:
Let T be a finite, nonempty subset of $\mathbb{R}^2$. Show that Clique can be computed in $O(|T| \log |T|)$ time:

$$\text{CLIQUE}(T) := \frac{1}{|T|-1} \sum_{\{(x,y),(x',y')\} \subseteq T} \left(|x - x'| + |y - y'|\right).$$

(4 points)

Exercise 7.3:
Show that, for quadratic netlength minimization, the Clique net model can be replaced equivalently by Star net model, by adjusting net weights, i.e. rather than minimizing the two programs in the form

$$\sum_{N \in \mathcal{N}} \frac{w(N)}{|N|-1} \sum_{\{p,q\} \subseteq N} \left(x(p) + x(\gamma(p)) - x(q) + x(\gamma(q))\right)^2,$$

(and its analogous in y-dimension), we can minimize

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$$\sum_{N \in \mathcal{N}} w'(N) \min_{x' \in \mathbb{R}} \sum_{p \in N} \left(x(p) + x(\gamma(p)) - x'\right)^2,$$

(and its analogous in y-dimension) for an appropriate weight function w'.

Conclude that for quadratic netlength minimization, it sufficies to solve a linear equation system $Ax = b$, where the number of nonzero entries of A can be bounded by a linear function in the number of pins and circuits.

(3+3 points)

Exercise 7.4:
Show that, for $m = 2$, the fractional relaxation of the MULTISECTION PROBLEM can be solved optimally in $O(n \log n)$ time, where n is the number of circuits.

(5 points)

Deadline: June 16th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss16/ss16.html`

In case of any questions feel free to contact me at saccardi@or.uni-bonn.de.