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Exercise Set 5

Exercise 5.1:
Let $[l_i, r_i] \subset \mathbb{R}$, $l_i \leq r_i$, $i = 1, \ldots, n$ be n intervals. Define

$$\begin{aligned} g(x) &:= |\{i \mid r_i \leq x\}| - |\{i \mid l_i \geq x\}| \qquad\qquad 1 \leq i \leq n; \\ l^* &:= \sup\{x \in \mathbb{R} \mid g(x) < 0\}; \\ r^* &:= \inf\{x \in \mathbb{R} \mid g(x) > 0\}. \end{aligned}$$

Describe an algorithm that computes l^* and r^* in $O(n)$ time.

Hint: you may use the fact that the k^{th} smallest number in a set of $m \geq k$ numbers can be computed in $O(m)$ time.

(4 points)

Exercise 5.2:
Let N be a finite set of pins, and let $\mathcal{S}(p)$ be a set of axis-parallel rectangles for each $p \in N$. We want to compute the *bounding box netlength* of N. To this end, we look for an axis-parallel rectangle R with minimum perimeter such that for every $p \in N$ there is a $S \in \mathcal{S}(p)$ with $R \cap S \neq \emptyset$. Let $n := \sum_{p \in N} |\mathcal{S}(p)|$; show that such a rectangle can be computed in $O(n^3)$ time.

(5 points)

Exercise 5.3:
Prove that the following problem is NP-complete for every constant $\alpha \geq 1$.

Input:	A set $\{[0, w_i] \times [0, h_i] \mid i = 1, \ldots, n\}$ of rectangular circuits and a rectangular chip area $[0, w] \times [0, h]$ s.t. $\alpha \sum_{i=1}^{n} w_i \, h_i \leq w \, h$.
Task:	Decide whether exists a feasible placement.

(4 points)

Exercise 5.4:
Consider the placement instance in Figure 1. Each of the circuits C_1, C_2, C_3, C_4, C_5, and C_6 must be placed in one of the three circuit rows. They must lie within the chip area and they must not intersect each other in the interior. The orientation of the circuits is fixed. The pins are point-shaped and their location is at the center of the labeled circles in figure; their location, relative

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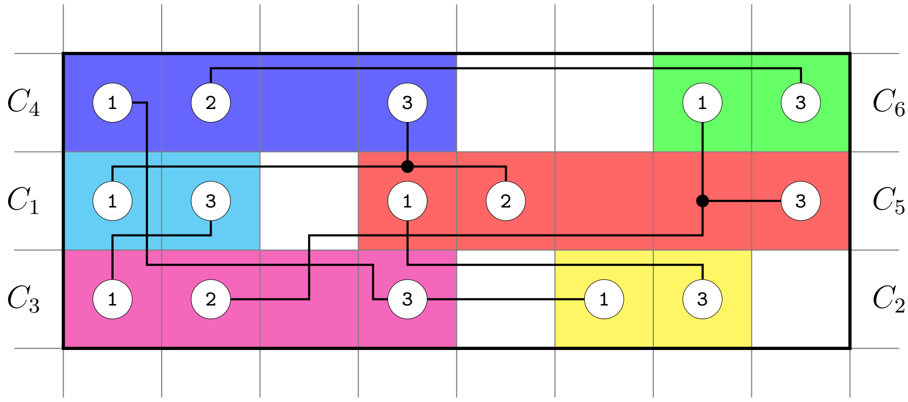

Figure 1: A feasible placement for the circuits $C_1, \ldots, C_6$. The chip area is given by the black rectangle. The labeled circles in the circuits mark the position of the pins; Steiner points are drawn as filled circles. The Steiner tree net length of this placement is 32.

to their parent circuit, is fixed. Nets are described by Steiner trees connecting the pins; these are not pairwise disjoint, therefore Steiner points are marked explicitly. The grid lines have unit distance.

Definition. *Let $\mathcal{N}$ denote the netlist. The* Steiner tree net length *of a placement is defined as*

$$\textsc{Steiner}(\mathcal{N}) := \sum_{N \in \mathcal{N}} \textsc{Steiner}(N).$$

(a) Prove that there is no feasible placement with $\textsc{Steiner}(\mathcal{N}) < 9$. Can you find a better lower bound?

(b) Determine a feasible placement of minimum Steiner net length.

Note: if your placement is feasible but worse than the optimum by k units, part (b) will be scored $\max\{0, 5 - k\}$.

(2+5 points)

Deadline: June 2nd, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss16/ss16.html`

In case of any questions feel free to contact me at saccardi@or.uni-bonn.de.