Exercise Set 4

Exercise 4.1:
Let Y be a Steiner tree for a terminal set T, with $|T| \geq 2$, in which all leaves are terminals. Let k be the number of full components of Y. Prove that

$$k = 1 + \sum_{t \in T} (|\delta_Y(t)| - 1) .$$

(3 points)

Exercise 4.2:
Let $T \subset \mathbb{R}^2$ be a finite set of terminals, and $S_1, \ldots, S_m$ be rectangular blockages. Set $S = S_1 \cup \cdots \cup S_m$, and let $\mathring{S}$ denote the interior of S. Let $L > 0$.

Definition. *A rectilinear Steiner tree Y for T is* reach-aware *if every connected component of the intersection $R \cap \mathring{S}$ has length at most L. We define the* Hanan grid induced by an instance $(T, S_1, \ldots, S_m)$ *as the usual Hanan grid associated to $T \cup \{l_i, r_i \mid i = 1, \ldots, m\}$, where l_i (resp. r_i) is the lower left (resp. upper right) corner of S_i.*

Prove or disprove: there is always a shortest reach-aware Steiner tree for T that is a subgraph of the Hanan grid induced by $(T, S_1, \ldots, S_m)$.

(4 points)

Exercise 4.3:

RSMT WITH WEIGHTED SUM OF ELMORE DELAYS	
Input:	A finite set $T \subset \mathbb{R}^2$, a root $r \in T$; source resistance R_0, resistance R_u and capacitance C_u per wire unit; weights $w_i > 0$ and downstream capacitance $\mathrm{downcap}(t_i)$ for each $t_i \in T \setminus \{r\}$.
Task:	Construct a rectilinear Steiner tree Y for T that minimizes $\sum_{t_i \in T \setminus \{r\}} w_i \, \mathrm{ELMORE}(r, t_i)$.

We define $\mathrm{ELMORE}(r, t_i)$ as

$$\mathrm{ELMORE}(r, t_i) := R_0 \,\mathrm{downcap}(r) + \sum_{e=(u,v) \in Y[r,t_i]} \mathrm{res}(e) \left(\frac{\mathrm{cap}(e)}{2} + \mathrm{downcap}(v) \right)$$

where $\mathrm{cap}(e) := C_u \, \ell_1(u, v)$ and $\mathrm{res}(e) = R_u \, \ell_1(u, v)$.

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(a) Prove that the Hanan grid for T contains an optimum solution to RSMT with Weighted Sum of Elmore Delays.

(b) Show that this does not hold if, instead of the weighted sum, we minimize $\max_{t_i \in T \setminus \{r\}} \textsc{Elmore}(r, t_i)$.

(4+4 points)

Exercise 4.4:

Definition. *For a finite set $V \subset \mathbb{R}^2$, the* ℓ_1-Voronoi diagram *consists of all the regions P_v, $v \in V$ defined as*

$$P_v := \left\{ x \in \mathbb{R}^2 \;\middle|\; \|x - v\|_1 = \min_{w \in V} \|x - w\|_1 \right\}.$$

The ℓ_1-Delaunay triangulation *of V is the graph (V, E) with*

$$E := \{\{v, w\}, v, w \in V : v \neq w, |P_v \cap P_w| > 1\}.$$

Assume that for any two elements of V the slope of the line segment connecting these is neither 1 nor -1.

(a) Show that the ℓ_1-Delaunay triangulation is a planar graph.

(b) Show how a rectilinear minimum spanning tree for V can be computed in $O(|V| \log |V|)$ time. You can use the fact that the Delaunay triangulation can be computed in $O(|V| \log |V|)$ time.

(c) Show that the ℓ_1-Delaunay triangulation is not necessarily planar without the requirement that the slope of any segment connecting two elements of V is neither 1 nor -1.

(2+2+1 points)

Deadline: May 24th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss16/ss16.html`

In case of any questions feel free to contact me at saccardi@or.uni-bonn.de.