Exercise Set 1

Exercise 1.1:
Let $n \in \mathbb{N}$ such that $\log_2(n) \in \mathbb{N}$ and let $+ : \{0,1\}^{2n} \to \{0,1\}^{n+1}$ be the addition function of two binary n-bit integers:

Input: $A_i, B_i \in \{0,1\}$, $i = 0, 1, \ldots, n-1$, representing the numbers
$$A = \sum_{i=0}^{n-1} 2^i \cdot A_i, \qquad B = \sum_{i=0}^{n-1} 2^i \cdot B_i.$$
Output: The binary representation of $A + B$.

Construct two netlists (one for each of the following conditions) realizing the function $+$ using a library containing *AND*s, *OR*s and *XOR*s, each with two inputs, such that:

(a) the number of used circuits is at most $5n$;
(b) the number of circuits on each path from an input pin to an output pin is at most $n + \log_2(n)$.

For both netlists derive formulas for the number of used circuits and the number of circuits on the longest path from an input pin to an output pin.

(6 points)

Exercise 1.2:
Prove or disprove: for every netlist with technology mapping there is a logically equivalent one that only contains:

(a) *NOR*s;
(b) *XOR*s;
(c) *NAND*s.

(4 points)

Exercise 1.3:
Let $n \in \mathbb{N}$, $n \geq 7$. Prove that there exists a boolean function $f : \{0,1\}^n \to \{0,1\}$ such that there exists no netlist realizing f with at most $\frac{2^{n-1}}{n}$ circuits, each with at most two inputs.

(4 points)

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Exercise 1.4:
Show that it is possible to solve ALL-CROSSINGS in time $O((n+k)\log n)$, where k is the number of intersection points:

ALL-CROSSINGS

Input: A set Q of n line segments in the plane such that:
(a) $\forall\, s_1, s_2 \in Q$, $|s_1 \cap s_2| \leq 1$;
(b) $\forall\, s_1, s_2, s_3 \in Q$, $s_1 \cap s_2 \cap s_3 = \emptyset$.

Task: Find all the intersection points between pairs of segments.

(6 points)

Deadline: April 21st, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss16/ss16.html`

In case of any questions feel free to contact me at saccardi@or.uni-bonn.de.