Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

Prof. Dr. Stefan Hougardy
Siad Daboul

Exercise Sheet 11

Problem 11.1. (3 points)
Describe a polynomial-time algorithm which optimally solves any instance of the TRAVELING SALESMAN PROBLEM that is the metric closure of a weighted tree.

Problem 11.2. (3 points)
Let c_0 be the value of an optimal solution of an instance of the METRIC TSP and c_1 the cost of a second-shortest tour. Prove $\frac{c_1 - c_0}{c_0} \leq \frac{2}{n}$.

(Note that the second-shortest tour might have the same length as the optimal tour.)

Problem 11.3. (1+4 points)
Assume $G = (V, E)$ is the complete graph with an euclidean embedding $\varphi : V \to \mathbb{R}^2$ such that there are no two parallel edges. Show that

(i) An optimum euclidean tour for G does not intersect itself.
(ii) Given any initial tour T it is possible to construct an intersection free tour T' that is shorter than T in polynomial time by using 2-opt exchanges.

Problem 11.4. (4+1 points)

(i) Show that every edge in a 3-regular graph is contained in an even number of Hamiltonian circuits.
(ii) Is ANOTHER HAMILTONIAN CIRCUIT for 3-regular graphs in P?

Please hand in your solutions on Tuesday, **July 5$^{\text{th}}$**, before the lecture.