Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

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Exercise Sheet 10

Problem 10.1. (4 points)
Show that the Contraction Lemma still holds in the case when edges with length larger than 0 are added between terminals. Hereby, parallel edges are allowed.

Problem 10.2. (4 points)
The Metric Bipartite Traveling Salesman Problem is the problem of finding a Hamiltonian circuit of minimum cost in a bipartite graph G with a nonnegative cost function c satisfying $c(\{a,b\}) + c(\{a',b\}) + c(\{a',b'\}) \geq c(\{a,b'\})$ for $\{a,b\}, \{a',b\}, \{a,b'\}$, $\{a',b'\} \in E(G)$.

Prove that for any k, if there is a k-factor approximation algorithm for the Metric Bipartite Traveling Salesman Problem, there is also a k-factor approximation algorithm for the Metric Traveling Salesman Problem.

Problem 10.3. (4 points)
Consider the following algorithm for the Symmetric Traveling Salesman Problem with triangle inequality:

Start with an arbitrary city $u \in V$. Find a shortest edge $e = \{u,v\} \in \binom{V}{2}$ connecting u to another city v. This yields a subtour $T = (u,v,u)$. Let $U := V \setminus \{u,v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in T.
(ii) Add w to T between two neighbouring nodes $i,j \in T$ (by deleting edge $\{i,j\}$ and connecting i and j with w), such that the cost of the new tour is minimized, i.e. find neighbouring $i,j \in T$ such that that $d(i,w) + d(w,j) - d(i,j)$ is minimum. Remove w from U afterwards.

Show that this is a 2-approximation algorithm.

Problem 10.4. (4 points)
Consider the restriction of the Traveling Salesman Problem to complete graphs in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for this problem.

Please hand in your solutions on Tuesday, **June 28$^{\textbf{th}}$**, before the lecture.