Problem 10.1. (4 points)
Show that the Contraction Lemma still holds in the case when edges with length larger than 0 are added between terminals. Hereby, parallel edges are allowed.

Problem 10.2. (4 points)
The Metric Bipartite Traveling Salesman Problem is the problem of finding a Hamiltonian circuit of minimum cost in a bipartite graph $G$ with a nonnegative cost function $c$ satisfying $c(\{a, b\}) + c(\{a', b\}) + c(\{a, b'\}) \geq c(\{a, b'\})$ for $\{a, b\}, \{a', b\}, \{a, b'\}, \{a', b'\} \in E(G)$.

Prove that for any $k$, if there is a $k$-factor approximation algorithm for the Metric Bipartite Traveling Salesman Problem, there is also a $k$-factor approximation algorithm for the Metric Traveling Salesman Problem.

Problem 10.3. (4 points)
Consider the following algorithm for the Symmetric Traveling Salesman Problem with triangle inequality:

Start with an arbitrary city $u \in V$. Find a shortest edge $e = \{u, v\} \in \binom{V}{2}$ connecting $u$ to another city $v$. This yields a subtour $T = (u, v, u)$. Let $U := V \setminus \{u, v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in $T$.
(ii) Add $w$ to $T$ between two neighbouring nodes $i, j \in T$ (by deleting edge $\{i, j\}$ and connecting $i$ and $j$ with $w$), such that the cost of the new tour is minimized, i.e. find neighbouring $i, j \in T$ such that $d(i, w) + d(w, j) - d(i, j)$ is minimum. Remove $w$ from $U$ afterwards.

Show that this is a 2-approximation algorithm.

Problem 10.4. (4 points)
Consider the restriction of the Traveling Salesman Problem to complete graphs in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for this problem.

Please hand in your solutions on Tuesday, June 28th, before the lecture.