Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

Prof. Dr. Stefan Hougardy
Siad Daboul

Exercise Sheet 9

Problem 9.1. (3+3 points)
Consider the following greedy algorithm for the GRAPH STEINER TREE PROBLEM: Given a graph $G = (V, E)$ with terminal set R and edge lengths $c\colon E \to \mathbb{R}_+$ we compute a minimum spanning tree $T = \mathrm{mst}(R)$ in the terminal distance graph $G_D(R)$. While there is some $v \in V(G) \setminus R$ with $c(\mathrm{mst}(R \cup \{v\})) < c(\mathrm{mst}(R))$ set $R := R \cup \{v\}$ and remove any non-terminals of degree ≤ 2 (in $\mathrm{mst}(R)$) from R. Return $\mathrm{mst}(R)$.

Suppose that $V \setminus R$ forms a stable set.

(i) Show that this algorithm is a $\frac{3}{2}$ approximation algorithm.
(ii) Show that this algorithm is no ρ approximation for any $\rho < \frac{3}{2}$.

Problem 9.2. (3 points)
Show that in Mehlhorn's algorithm replacing the edges of the minimum spanning tree by corresponding shortest paths does not result in cycles.

Note: You may use that the Voronoi regions are computed with Dijkstra's algorithm.

Problem 9.3. (2+2+3 points)
Consider an instance $G = (V, E)$ of the STEINER TREE PROBLEM with terminal set R and edge lengths $c\colon E \to \mathbb{R}_+$. Denote the full components of an optimum k-Steiner tree $\mathrm{SMT}_k(R)$ with $T_1^*, \ldots, T_k^*$.

(i) Suppose that $V \setminus R$ forms a stable set. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_k^*)) .$$

(ii) Suppose that all shortest paths between any two vertices in G have length 1 or 2. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_k^*)) .$$

(iii) Show that the Loss Contraction Algorithm achieves an approximation ratio of $1.279 \cdot r_k$ in both cases.

Please hand in your solutions on Tuesday, **June 21st**, before the lecture.