Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

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Exercise Sheet 8

Problem 8.1. (4 points)
Let $T = \{t_1, t_2, \ldots, t_n\} \subset \mathbb{R}^2$ be a finite set of terminals in the rectilinear plane. Let $x, y : T \to \mathbb{R}$ be the projection on the first and second coordinate. Show that if $x(t_i) < x(t_{i+1})$ for $i = 1, \ldots, n-1$ and $|\{y(t) : t \in T\}| = k$ where k is some fixed constant, the Steiner Tree Problem can be solved in polynomial time.

Hint: You may use without a proof that every steiner point is of the form $(x(t_i), y(t_j))$ for some $i, j \in \{1, \ldots, n\}$.

Problem 8.2. (4 points)
Consider the restriction $\mathcal{P}$ of the unweighted Vertex Cover Problem to graphs where the maximum degree of every vertex is bounded by a constant B.

Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio $1+\epsilon$, then there exists a polynomial time approximation algorithm for problem $\mathcal{P}$ with performance ratio $1 + (B+1)\epsilon$.

Problem 8.3. (4 points)
Consider the following algorithm for the Graph Steiner Tree Problem with 3 terminals v_1, v_2, and v_3: Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z minimizing $\sum_{i=1}^{3} \mathrm{dist}(v_i, z)$ under the conditions

(i) $\mathrm{dist}(v_i, z) \le \mathrm{dist}(v_1, v_2)$ for $i \in \{1, 2\}$ and
(ii) $\mathrm{dist}(v_3, z) \le a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm can be implemented in $\mathcal{O}(m + n \log n)$ and works correctly.

Problem 8.4. (4 points)
Give an $\mathcal{O}(n^3 t^2)$ algorithm for the Steiner Tree Problem in planar graphs with all terminals lying on the outer face, where n is the number of vertices and t the number of terminals.

Hint: Modify the Dreyfus-Wagner algorithm.

Please hand in your solutions on Tuesday, **June 14th**, before the lecture.