Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

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Exercise Sheet 7

Problem 7.1. (4 points)
For every fixed $d \in \mathbb{N}$ the d-DIMENSIONAL BIN PACKING problem is defined as follows: Given item sizes $a_1, \ldots, a_n \in \mathbb{R}^d_{\geq 0}$ find some $k \in \mathbb{N}$ and an assignment $f : \{1, \ldots, n\} \to \{1, \ldots, k\}$ such that $\sum_{i:f(i)=j} a_i \leq \mathbb{1}_d$ for all $j = 1, \ldots, k$. Here $\mathbb{1}_d$ denotes the d dimensional vector that is 1 in every coordinate.

Show that for any $\epsilon > 0$ there is an asymptotic $d + \epsilon$ approximation algorithm, i.e. a polynomial algorithm that always finds a solution with at most $(d + \epsilon)\mathrm{OPT} + C_\epsilon$ bins where C_ϵ is some constant depending on ϵ.

Problem 7.2. (4 points)
Given an undirected graph $G = (V, E)$, a partition $V = A \dot{\cup} B$ and edge costs $c : E \to \mathbb{R}_{\geq 0}$ we are looking for a graph $G' = (V', E')$, where $V' = V, E' \subseteq E$, minimizing $\sum_{e \in E(G')} c(e)$ such that for every $w \in B$ there is some $v \in A$ and a v-w path in G'.

Show that there is a polynomial algorithm which solves this problem optimally.

Problem 7.3. (4+4 points)
In this exercise we will derive a polynomial algorithm for a variant of the BIN PACKING problem where we relax the bin capacities by some $\epsilon > 0$.

(i) Give an algorithm for BIN PACKING restricted to instances with a constant number of different item sizes whose running time is polynomially bounded in the number n of items.

Hint: Use dynamic programming.

(ii) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which for any instance $I = (a_1, \ldots, a_n)$ of the BIN PACKING problem finds a packing using the optimum number of bins but possibly violating the capacity constraints by ε, i. e. an $f : \{1, \ldots, m\} \to \{1, \ldots, \mathrm{OPT}(I)\}$ with $\sum_{f(i)=j} a_i \leq 1 + \varepsilon$ for all $j \in \{1, \ldots, \mathrm{OPT}(I)\}$.

Please hand in your solutions on Tuesday, **June 7th**, before the lecture.