Research Institute for Discrete Mathematics
Prof. Dr. Stefan Hougardy
Siad Daboul
Approximation Algorithms
Summer Term 2016

# Exercise Sheet 6

**Problem 6.1.** (4 points)
Let $A = (a_i)_{1 \le i \le p}$ and $B = (b_j)_{1 \le j \le q}$ be two inputs of the BIN PACKING problem. We write $A \subseteq B$ if there are indices $1 \le k_1 < k_2 < \cdots < k_p \le q$ with $a_i = b_{k_i}$ for $1 \le i \le p$. An algorithm for the BIN PACKING problem is called monotone if for inputs $A$ and $B$ with $A \subseteq B$ the algorithm needs at least as many bins for $B$ as for $A$. Show:

(i) NEXT FIT is monotone.

(ii) FIRST FIT is not monotone.

**Problem 6.2.** (4 points)
Show that if all item sizes $a_1, \ldots, a_n$ satisfy $a_i > \frac{1}{3}$ then BIN PACKING can be solved optimally in time $\mathcal{O}(n \log n)$.

**Problem 6.3.** (4 points)
Consider the following scheduling problem: Given a set of jobs $(s_1, d_1, l_1), \ldots, (s_n, d_n, l_n)$ (where all tuples are nonnegative) we are looking for a permutation $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ and job start times $t : \{1, \ldots, n\} \to [0, \infty)$ such that $0 \le s_{\pi(i)} \le t(\pi(i)) \le d_{\pi(i)}$ for $i = 1, \ldots, n$ and $t(\pi(i+1)) \ge t(\pi(i)) + l_{\pi(i)}$ for $i = 1, \ldots, n-1$ such that $t(\pi(n)) + l_{\pi(n)}$ is minimal.

Show that this problem is strongly NP hard.

**Problem 6.4.** (4 points)
Show that if all item sizes are of the form $a_i = k \cdot 2^{-b_i}$ for some $b_i \in \mathbb{N}, i = 1, \ldots, n$ and some fixed $k \in \mathbb{N}$ then the FIRST FIT DECREASING algorithm always finds an optimum solution.

Please hand in your solutions on Tuesday, **May 31st**, before the lecture.