Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

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Exercise Sheet 4

Problem 4.1. (3 points)
Consider the following procedure for unweighted MINIMUM VERTEX COVER: Given a graph G, compute a DFS tree for every connected component. Return all vertices with non-zero out-degree in the tree. Show that this is a 2-approximation algorithm.

Problem 4.2. (4+3 points)
The LP relaxation of the MINIMUM WEIGHT VERTEX COVER PROBLEM is

$$\min\{cx : M^T x \geq 1, x \geq 0\},$$

where $M \in \{0,1\}^{n\times m}$ is the incidence matrix of an undirected graph G and $c \in \mathbb{R}^n_+$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

(i) Show that the above LP relaxation always has a half-integral optimum solution.

(ii) Given a graph $G = (V, E)$ with weights $c \in \mathbb{R}^n_+$ and a coloring $\varphi : V \to \{1, \ldots, k\}$ show how the LP relaxation can be used to find a vertex cover $X \subseteq V$ with cost $c(X) \leq (2 - \frac{2}{k})\mathrm{OPT}$. Here OPT denotes the cost of an optimum solution.

Problem 4.3. (3 points)
Consider the following variant of the SET COVER PROBLEM: Given a set $\mathcal{U}$, sets $\mathcal{S} = \{S_1, \ldots, S_m\}$ where $\cup_{i=1}^m S_i = \mathcal{U}$ and some $k \in \mathbb{N}$ find k sets $S_{i_1}, \ldots, S_{i_k} \in \mathcal{S}$ such that $|\cup_{j=1,\ldots,k} S_{i_j}|$ is maximum. Show that iteratively picking the element that maximizes the amount of not yet covered elements is a $1 - \frac{1}{e}$ approximation.

Problem 4.4. (3 points)
Show that any 4-colorable graph with n vertices can be colored with $\mathcal{O}(n^{\frac{2}{3}})$ colors in polynomial time.

Please hand in your solutions on Tuesday, **May 10$^{\text{th}}$**, before the lecture.
Note that **there will be no exercise class** on Thursday, **May 5$^{\text{th}}$**.