Exercise Sheet 3

Problem 3.1. (4 points)
Prove that the following is a 2-approximation algorithm for the $k$-CENTER PROBLEM:
Given an instance $(X, \text{dist}, k)$, start with $C = \{x\}$ for $x \in X$ arbitrary. Successively add some $x \in X \setminus C$ to $C$ maximizing $\min_{c \in C} \text{dist}(x, c)$ until $|C| = k$. Return $C$.

Problem 3.2. (4 points)
An instance of MAXIMUM SATISFIABILITY is called $k$-satisfiable if any $k$ of its clauses can be satisfied simultaneously. Give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a $\sqrt{5} - 1$-fraction of the clauses.

Problem 3.3. (4 points)
Consider the DIRECTED STEINER TREE PROBLEM: Given an edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at $r$ that contains every vertex in $T$.

Show that a $k$-approximation algorithm for the DIRECTED STEINER TREE PROBLEM can be used to obtain a $k$-approximation algorithm for SET COVER.

Problem 3.4. (4 points)
Show that the following BOUNDED MINIMUM CUT problem is NP-complete: Given an undirected graph $G = (V, E)$, two vertices $s, t \in V$ and two numbers $k, b \in \mathbb{N}$. Is there a partition $V = V_1 \cup V_2$ with $s \in V_1, t \in V_2, \max_{i \in \{1,2\}} |V_i| \leq k$ and $|\delta(V_i)| \leq b$?

Please hand in your solutions on Tuesday, May 3rd, before the lecture.