Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

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Exercise Sheet 3

Problem 3.1. (4 points)
Prove that the following is a 2-approximation algorithm for the k-CENTER PROBLEM: Given an instance (X, dist, k), start with $C = \{x\}$ for $x \in X$ arbitrary. Successively add some $x \in X \setminus C$ to C maximizing $\min_{c \in C} \mathrm{dist}(x, c)$ until $|C| = k$. Return C.

Problem 3.2. (4 points)
An instance of MAXIMUM SATISFIABILITY is called k-satisfiable if any k of its clauses can be satisfied simultaneously. Give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a $\frac{\sqrt{5}-1}{2}$-fraction of the clauses.

Problem 3.3. (4 points)
Consider the DIRECTED STEINER TREE PROBLEM: Given a edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at r that contains every vertex in T.

Show that a k-approximation algorithm for the DIRECTED STEINER TREE PROBLEM can be used to obtain a k-approximation algorithm for SET COVER.

Problem 3.4. (4 points)
Show that the following BOUNDED MINIMUM CUT problem is NP-complete: Given an undirected graph $G = (V, E)$, two vertices $s, t \in V$ and two numbers $k, b \in \mathbb{N}$. Is there a partition $V = V_1 \dot{\cup} V_2$ with $s \in V_1, t \in V_2, \max_{i \in \{1,2\}} |V_i| \leq k$ and $|\delta(V_1)| \leq b$?

Please hand in your solutions on Tuesday, **May 3rd**, before the lecture.