Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2016

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Exercise Sheet 2

Problem 2.1. (4 points)
Show that the following problem is NP-complete:
Given an undirected graph $G = (V, E)$ and some $k \in \mathbb{N}$ is there a partition $V = V_1 \dot{\cup} V_2 \dot{\cup} \dots \dot{\cup} V_k$ such that for all $i = 1, \dots, k$ and $v \in V_i$ we have $|\delta_{G[V_i]}(v)| = 1$ (i.e. all $G[V_i]$ are perfect matchings)?

Problem 2.2. (4+2 points)
Consider the following algorithm for the Maximum Cut problem: Given an undirected graph $G = (V, E)$, find a set $X \subseteq V$ maximizing $|\delta(X)|$. Start with $X = \emptyset$. If adding a single vertex to X or deleting a single vertex from X makes $|\delta(X)|$ larger, then do so. Repeat until no improvement is possible.

(i) Show that the algorithm is a polynomial $\frac{1}{2}$-factor approximation algorithm.
(ii) Does the algorithm always find an optimum solution for planar graphs, or for bipartite graphs?

Problem 2.3. (4+2 points)
Let $G = (V, E)$ be an undirected graph. The chromatic number $\chi(G) \in \mathbb{N}$ denotes the minimum amount of colors such that G can be $\chi(G)$ colored.

(i) Show that for any orientation $G^{\leftrightarrow}$ of G the chromatic number satisfies

$$\chi(G) \leq \max_{P \text{ path in } G^{\leftrightarrow}} |V(P)|.$$

(ii) Show that there is equality in (i) for some orientation $G^{\leftrightarrow}$ of G.

Please hand in your solutions on Tuesday, **April 26$^{\text{th}}$**, before the lecture.