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Exercise Sheet 1

Definition.
Let P be a maximization problem with weight function $w : \mathcal{X} \to \mathbb{R}^+$ where $\mathcal{X}$ is the set of feasible solutions. If an algorithm A always returns a solution x with cost

$$w(x) \geq \tau \sup_{y \in \mathcal{X}} w(y)$$

for some $\tau > 0$ we call A a *τ-approximation algorithm*. For minimization problems we require $w(x) \leq \tau \inf_{y \in \mathcal{X}} w(y)$ instead.

Problem 1.1. (2+2+4 points)
Show that there is a linear-time $\frac{1}{2}$-approximation algorithm for the problems "i", "ii" and a linear-time 2-approximation algorithm for problem "iii".

(i) Given a digraph G with non-negative edge weights, find an acyclic subgraph of maximum weight.

(ii) Given an undirected, unweighted graph G, find vertices $v, w \in V(G)$ such that their distance is maximum.

(iii) Given a directed cycle $C = (V, E)$ and a set of undirected edges $E_1 \subseteq \{\{v, w\} | v \neq w, v \in V, w \in V\}$ we are looking for an orientation $E_1^{\leftrightarrow}$ of E_1 such that, in the digraph $G' = (V, E \cup E_1^{\leftrightarrow})$, $\max_{e \in E} |\{C' \text{ directed cycle} | e \in E(C'), |E(C') \cap E_1^{\leftrightarrow}| = 1\}|$ is minimal.

Problem 1.2. (4 points)
Show that if there is a polynomial $\frac{1}{2}$-approximation algorithm for the maximum stable set problem, then there is also a polynomial $(1 - \epsilon)$-approximation algorithm for every $\epsilon > 0$.

Problem 1.3. (4 points)
Show that the following problem is NP-complete:
Given a digraph $G = (V, E)$, is there some $X \subset V$ such that $E(G[X]) = \emptyset$ and that for all $v \in V \setminus X$ we have $\delta^+_{G[X \cup \{v\}]}(v) \neq \emptyset$.
Hint: Use a reduction from Satisfiability.

Please hand in your solutions in **groups of (up to) 2 students** on Tuesday, **April 19$^{\text{th}}$**, before the lecture.