Chip Design
Summer term 2015

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Exercise Set 11

Exercise 11.1:
Show that the VERTEX-DISJOINT PATHS PROBLEM is *NP*-complete even if G is a subgraph of a track graph G_T with two routing planes.
Note: This means that G_T has vertex set $\{1,\ldots,n_x\} \times \{1,\ldots,n_y\} \times \{1,2\}$ for some $n_x, n_y \in \mathbb{N}$ and edge set $\{\{(x,y,z),(x',y',z')\} : |x-x'|z + |y-y'|(3-z) + |z-z'| = 1\}$.

Hint: Consider the proof of Theorem 5.2.

(5 points)

Exercise 11.2:
Formulate the SIMPLE GLOBAL ROUTING PROBLEM as an integer linear program with a polynomial number of variables and constraints.

(5 points)

Exercise 11.3:
Show that for every $\delta > 0$ there exists a $\gamma > 0$ such that the following is true. Let $(G,u,l,\mathcal{N},c,w)$ be an instance of the SIMPLE GLOBAL ROUTING PROBLEM with $u(e) \geq \gamma \ln|E(G)|$ and $w(N,e) = 1$ for all $e \in E(G)$ and $N \in \mathcal{N}$, and let a feasible fractional solution with weighted net length L be given. Show that then there exists an integral solution for the modified instance in which all capacities are multiplied by $1+\delta$, such that the weighted net length is at most $(1+\delta)L$.

(5 points)

Exercise 11.4:
Given an instance of the MIN-MAX RESOURCE SHARING PROBLEM with σ-optimal block solvers for some fixed $\sigma \geq 1$. Prove that a $\sigma(1+\omega)$-approximate solution can be computed in in $O(\theta \log|\mathcal{R}|((|\mathcal{N}| + |\mathcal{R}|)\log\log|\mathcal{R}| + \min\{\rho|\mathcal{N}|, |\mathcal{N}| + |\mathcal{R}'|\}\omega^{-2}))$ time, where $\rho := \max\{1, \sup\{b_r/\lambda^* | r \in \mathcal{R}, N \in \mathcal{N}, b \in \mathcal{B}_N\}\}$ and $\mathcal{R}' := \{r \in \mathcal{R} | \exists N \in \mathcal{N}, b \in \mathcal{B}_N \text{ with } b_r > \lambda^*\}$.
Remark: For practical instances ρ and $|\mathcal{R}'|$ are often small.

(5 points)

Deadline: Thursday, July 9th, before the lecture.

The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss15/ss15.html

In case of any questions feel free to contact me at ahrens@or.uni-bonn.de.