

Exercise Set 6

Exercise 6.1:

Recall the LP for the d -dimensional arrangement problem from the lecture for $d = 2$:

$$\begin{array}{ll}
 \min & \sum_{e \in E(G)} w(e)l(e) \\
 \text{s.t.} & \sum_{y \in X} l(\{x, y\}) \geq \frac{1}{4}(|X| - 1)^{1+1/2} \quad \forall X \subseteq V(G), \forall x \in X \\
 & l(\{x, y\}) + l(\{y, z\}) \geq l(\{x, z\}) \quad \forall x, y, z \in V(G) \\
 & l(\{x, y\}) \geq 1 \quad \forall x, y \in V(G), x \neq y \\
 & l(\{x, x\}) = 0 \quad \forall x \in V(G)
 \end{array}$$

Let L be the optimal value of this LP. Show that there is no feasible solution of the given instance of the 2-dimensional arrangement problem with cost less than L .

(5 points)

Exercise 6.2:

The GRIDDED PLACEMENT PROBLEM is an extension of the STANDARD PLACEMENT PROBLEM with a grid $\Gamma = \Gamma_x \times \Gamma_y$, $\Gamma_x := \{k \cdot \delta_x : k \in \mathbb{Z}\}$ and $\Gamma_y := \{k \cdot \delta_y : k \in \mathbb{Z}\}$ for some $\delta_x, \delta_y \in \mathbb{Z}$, where the lower left corner of each circuit is required to be located on one of the grid points. Prove: The GRIDDED PLACEMENT PROBLEM is NP -hard even if an optimum solution of the associated ungridded placement problem is known.

(4 points)

Exercise 6.3:

Given a finite set $T \subseteq \mathbb{R}^2$, show how

$$\text{CLIQUE}(T) := \frac{1}{|T| - 1} \sum_{(x,y), (x',y') \in T} (|x - x'| + |y - y'|)$$

can be computed in $\mathcal{O}(|T| \log |T|)$ time.

(3 points)

Exercise 6.4:

Show that for quadratic netlength minimization CLIQUE net models can be replaced equivalently by STAR net models by adjusting net weights.

Conclude that for quadratic netlength minimization it suffices to solve a linear equation system $Ax = b$, where the number of non-zero entries of A can be bounded by a linear function in the number of pins and circuits.

(4 points)

Exercise 6.5:

Prove: The STANDARD PLACEMENT PROBLEM with only one circuit and without blockages can be solved in $O(|P|)$ time, where P is the set of pins.

Note: You can use that the following problem can be solved in $O(m)$ time: Given a set of m numbers (x_1, \dots, x_m) and associated positive weights (w_1, \dots, w_m) , find $i \in \{1, \dots, m\}$ minimizing $\sum_{j \in \{1, \dots, m\}} w_j |x_i - x_j|$.

(4 points)

Deadline: Tuesday, June 2nd, before the lecture.

The websites for lecture and exercises are linked at

<http://www.or.uni-bonn.de/lectures/ss15/ss15.html>

In case of any questions feel free to contact me at ahrens@or.uni-bonn.de.