Chip Design
Summer term 2015

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Exercise Set 3

Exercise 3.1:
Let N be an instance of the Rectilinear Steiner Tree Problem and $r \in N$. For a rectilinear Steiner tree Y we denote by $f(Y)$ the maximum length of a path from r to any element of $N \backslash r$ in Y.

a) Describe an instance in which no shortest Steiner tree minimizes $f(Y)$ and no Steiner tree minimizing $f(Y)$ is shortest.

b) Consider the problem of finding a shortest Steiner tree Y minimizing $f(Y)$ among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid?

(1+4 points)

Exercise 3.2:
Let Y be a Steiner tree for terminal set T (with $|T| \geq 2$) in which all leaves are terminals. Prove that $\sum_{t \in T} \Big(|\delta_Y(t)| - 1 \Big) = k - 1$, where k is the number of full components of Y.

(4 points)

Exercise 3.3:
For a finite nonempty set $T \subset \mathbb{R}^2$ we define

$$\begin{aligned}
\mathrm{BB}(T) &:= \max_{(x,y)\in T} x - \min_{(x,y)\in T} x + \max_{(x,y)\in T} y - \min_{(x,y)\in T} y \\
\textsc{Steiner}(T) &:= \text{length of a shortest rectilinear Steiner tree for } T \\
\mathrm{MST}(T) &:= \text{length of a minimum spanning tree in the complete graph on } T, \\
& \text{ where edge weights are } l_1\text{-distances}
\end{aligned}$$

Prove that

(a) $\mathrm{BB}(T) \leq \textsc{Steiner}(T) \leq \mathrm{MST}(T)$ for all finite sets $T \subset \mathbb{R}^2$.

(b) $\textsc{Steiner}(T) = \mathrm{BB}(T)$ for all $T \subset \mathbb{R}^2$ with $|T| \leq 3$

(c) $\textsc{Steiner}(T) \leq \frac{3}{2} \cdot \mathrm{BB}(T)$ for all $T \subset \mathbb{R}^2$ with $|T| \leq 5$

(d) there exists no $k \in \mathbb{R}$ with $\textsc{Steiner}(T) \leq k \cdot \mathrm{BB}(T)$ for all finite sets $T \subset \mathbb{R}^2$

(1+1+3+2 points)

Exercise 3.4:
Let $n \in \mathbb{N}$ be a number and $\{[l_i, r_i]\}_{i \in \{1,\dots,n\}}$ be a set of line segments (i.e. $l_i \leq r_i$ holds for $i \in \{1, \dots, n\}$). We define

$$g(x) := |\{i \in \{1, \dots, n\} \,|\, r_i \leq x\}| - |\{i \in \{1, \dots, n\} \,|\, l_i \geq x\}|.$$

Describe an algorithm that computes $l^* := \sup\{x \in \mathbb{R} | g(x) < 0\}$ and $r^* := \inf\{x \in \mathbb{R} | g(x) > 0\}$ in $O(n)$ time.
Note: You can use that the kth smallest number in a set of $m \geq k$ numbers can be computed in $O(m)$ time.

(4 points)

Deadline: Thursday, April 30th, before the lecture.

The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss15/ss15.html

In case of any questions feel free to contact me at ahrens@or.uni-bonn.de.