Chip Design
Summer term 2015

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# Exercise Set 1

**Exercise 1:**
Let $n \in \mathbb{N}$ such that $\log_2(n) \in \mathbb{N}$ and let $+ : \{0,1\}^{2n} \to \{0,1\}^{n+1}$ be the addition function of two binary $n$-bit integers:

> **Input:** $A_i$, $B_i \in \{0,1\}$ for $i = 0, 1, \ldots, n-1$ representing $A = \sum_{i=0}^{n-1} 2^i \cdot A_i$ and $B = \sum_{i=0}^{n-1} 2^i \cdot B_i$.
>
> **Output:** The binary representation of $A + B$.

Construct two netlists (one for condition a) and one for condition b)) realizing the function $+$ using a library containing *AND*s, *OR*s and *XOR*s such that

a) The number of used circuits is at most $5n$.

b) The number of circuits on each path from an input pin to an output pin is at most $n + \log_2(n)$.

For both netlists derive formulas for the number of used circuits and the number of circuits on the longest path from an input pin to an output pin.

(8 points)

**Exercise 2:**
Prove or disprove: For every netlist with technology mapping there is a logically equivalent one that only contains a) *NOR*s b) *XOR*s c) *NAND*s.

(7 points)

**Exercise 3:**
Let $N$ be a finite set of pins, and let $\mathcal{S}(p)$ be a set of axis-parallel rectangles for each $p \in N$. We want to compute the bounding box net length of $N$. To this end, we look for an axis-parallel rectangle $R$ with minimum perimeter such that for every $p \in N$ there is a $S \in \mathcal{S}(p)$ with $R \cap S \neq \emptyset$. Let $n := \sum_{p \in N} |\mathcal{S}(p)|$.
Show that such a rectangle can be computed in $O(n^3)$ time.

Hint: Enumerate possible coordinates for the lower left corner of $R$.

(5 points)

**Deadline:** Thursday, April 16th, before the lecture.

The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss15/ss15.html

In case of any questions feel free to contact me at ahrens@or.uni-bonn.de.