Exercise Set 1

Exercise 1:
Let \( n \in \mathbb{N} \) such that \( \log_2(n) \in \mathbb{N} \) and let \( + : \{0,1\}^{2n} \to \{0,1\}^{n+1} \) be the addition function of two binary \( n \)-bit integers:

**Input:** \( A_i, B_i \in \{0,1\} \) for \( i = 0, 1, \ldots, n-1 \) representing \( A = \sum_{i=0}^{n-1} 2^i \cdot A_i \)
and \( B = \sum_{i=0}^{n-1} 2^i \cdot B_i \).

**Output:** The binary representation of \( A + B \).

Construct two netlists (one for condition a) and one for condition b) realizing the function + using a library containing ANDs, ORs and XORs such that

a) The number of used circuits is at most \( 5n \).

b) The number of circuits on each path from an input pin to an output pin is at most \( n + \log_2(n) \).

For both netlists derive formulas for the number of used circuits and the number of circuits on the longest path from an input pin to an output pin.

(8 points)

Exercise 2:
Prove or disprove: For every netlist with technology mapping there is a logically equivalent one that only contains a) NORs b) XORs c) NANDs.

(7 points)

Exercise 3:
Let \( N \) be a finite set of pins, and let \( S(p) \) be a set of axis-parallel rectangles for each \( p \in N \). We want to compute the bounding box net length of \( N \). To this end, we look for an axis-parallel rectangle \( R \) with minimum perimeter such that for every \( p \in N \) there is a \( S \in S(p) \) with \( R \cap S \neq \emptyset \). Let \( n := \sum_{p \in N} |S(p)| \).

Show that such a rectangle can be computed in \( O(n^3) \) time.

Hint: Enumerate possible coordinates for the lower left corner of \( R \).

(5 points)

Deadline: Thursday, April 16th, before the lecture.

The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss15/ss15.html

In case of any questions feel free to contact me at ahrens@or.uni-bonn.de.