Summer Term 2015 | Research Institute for Discrete Mathematics
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Approximation Algorithms

Exercise Sheet 9

Exercise 9.1:
Let T be a minimum-weight spanning tree in a weighted undirected graph G with non-negative edge weights. Let G' arise from G by either

(i) adding a new vertex s to G and connecting s to some of the vertices in G by non-negative weighted edges or

(ii) adding a new non-negative weighted edge e to G.

Show how a minimum-weight spanning tree for G' can be computed in linear time in both cases.

(4 points)

Exercise 9.2:
Show that in Mehlhorn's algorithm replacing the edges of the minimum spanning tree by corresponding shortest paths does not result in cycles.

(3 points)

Exercise 9.3:
Show that the Contraction Lemma still holds in the case when edges of length larger than 0 are added between terminals. Hereby, parallel edges are allowed.

(4 points)

Exercise 9.4:
Consider an instance $G = (V, E)$ of the Steiner Tree Problem with terminal set R and edge lengths $c \colon E \to \mathbb{R}_+$. Denote the full components of an optimum k-Steiner tree $SMT_k(R)$ with $T_1^*, \ldots, T_k^*$.

(i) Suppose that $V \backslash R$ forms a stable set. Show that

$$mst(R) \leq 2 \cdot (smt_k(R) - loss(T_1^*, \ldots, T_k^*)) \, .$$

(ii) Suppose that all shortest paths between any two vertices in T have length 1 or 2. Show that

$$mst(R) \leq 2 \cdot (smt_k(R) - loss(T_1^*, \ldots, T_k^*)) \, .$$

(iii) Show that the Loss Contraction Algorithm achieves an approximation ratio of $1.279 \cdot r_k$ in both cases.

(5 points)

Please turn in your solutions on Tuesday, **June 23rd**, before the lecture.