Summer Term 2015 Research Institute for Discrete Mathematics
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Approximation Algorithms

Exercise Sheet 8

Exercise 8.1:
Give an $\mathcal{O}(n^3t^2)$ algorithm for the Steiner Tree Problem in planar graphs with all terminals lying on the outer face, where n is the number of vertices and t the number of terminals.

Hint: Modify the Dreyfus-Wagner algorithm.

(4 points)

Exercise 8.2:

(i) Show that the algorithm of Kou, Markowsky, and Berman is a $(2 - \frac{2}{t})$-factor approximation, where t is the number of terminals.

(ii) Show that the analysis in (i) is tight.

(3 + 1 points)

Exercise 8.3:
Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender.

(i) Prove that the restriction of this problem to instances with $S \cup R = V$ is in P.

(ii) Prove that the restriction of this problem to instances with $S \cup R \subsetneq V$ is NP-hard and give a 2-factor approximation algorithm.

(2 + 3 points)

Exercise 8.4:
Consider the restriction $\mathcal{P}$ of the unweighted Vertex Cover Problem to graphs where the maximum degree of every vertex is bounded by a constant B.
Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio $1 + \epsilon$, then there exists a polynomial time approximation algorithm for problem $\mathcal{P}$ with performance ratio $1 + (B + 1)\epsilon$.

(3 points)

Please turn in your solutions on Tuesday, **June 16th**, before the lecture.