Summer Term 2015
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Approximation Algorithms

Exercise Sheet 7

Exercise 7.1:

(i) Give an algorithm for BIN PACKING with a constant number of different item sizes whose running time is polynomially bounded in the number n of items.

(ii) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which for any instance $I = (a_1, \ldots, a_n)$ of BIN PACKING finds a packing using the optimum number of bins but possibly violating the capacity constraints by ε, i. e. an assignment $f : \{1, \ldots, n\} \to \{1, \ldots, \mathrm{OPT}(I)\}$ with $\sum_{f(i)=j} a_i \leq 1 + \varepsilon$ for all $j \in \{1, \ldots, \mathrm{OPT}(I)\}$.

(iii) Show that the MULTIPROCESSOR SCHEDULING PROBLEM (see Exercise 6.3) has an approximation scheme.

(4 + 4 + 3 points)

Exercise 7.2:
Consider the following algorithm for the GRAPH STEINER TREE PROBLEM with 3 terminals v_1, v_2, and v_3: Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z minimizing $\sum_{i=1}^{3} dist(v_i, z)$ under the conditions

- $dist(v_i, z) \leq dist(v_1, v_2)$ for $i \in \{1, 2\}$ and
- $dist(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm can be implemented in $\mathcal{O}(m + n \log n)$ and works correctly.

(3 points)

Exercise 7.3:
Let $\mathcal{P}$ be an optimization problem. An algorithm A for $\mathcal{P}$ is an *absolute approximation algorithm* for $\mathcal{P}$ if there is a constant $k \in \mathbb{N}$ such that

$$|A(I) - \mathrm{OPT}| \leq k$$

holds for any instance I of $\mathcal{P}$.
Prove that (provided $P \neq NP$) there is no absolute approximation algorithm for the GRAPH STEINER TREE PROBLEM.

(2 points)

Please turn in your solutions on Tuesday, **June 9th**, before the lecture.