Approximation Algorithms

Exercise Sheet 7

Exercise 7.1:

(i) Give an algorithm for Bin Packing with a constant number of different item sizes whose running time is polynomially bounded in the number $n$ of items.

(ii) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which for any instance $I = (a_1, \ldots, a_n)$ of Bin Packing finds a packing using the optimum number of bins but possibly violating the capacity constraints by $\varepsilon$, i.e., an assignment $f : \{1, \ldots, n\} \to \{1, \ldots, \text{OPT}(I)\}$ with $\sum_{f(i) = j} a_i \leq 1 + \varepsilon$ for all $j \in \{1, \ldots, \text{OPT}(I)\}$.

(iii) Show that the Multiprocessor Scheduling Problem (see Exercise 6.3) has an approximation scheme.

(4 + 4 + 3 points)

Exercise 7.2:

Consider the following algorithm for the Graph Steiner Tree Problem with 3 terminals $v_1$, $v_2$, and $v_3$: Find a shortest path $P$ between $v_1$ and $v_2$ and let $a$ be the distance of $v_3$ to $P$. Then find a vertex $z$ minimizing $\sum_{i=1}^{3} \text{dist}(v_i, z)$ under the conditions

- $\text{dist}(v_i, z) \leq \text{dist}(v_1, v_2)$ for $i \in \{1, 2\}$ and
- $\text{dist}(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from $z$ to the terminals. Show that the algorithm can be implemented in $O(m + n \log n)$ and works correctly.

(3 points)

Exercise 7.3:

Let $\mathcal{P}$ be an optimization problem. An algorithm $A$ for $\mathcal{P}$ is an absolute approximation algorithm for $\mathcal{P}$ if there is a constant $k \in \mathbb{N}$ such that

$|A(I) - \text{OPT}| \leq k$

holds for any instance $I$ of $\mathcal{P}$.

Prove that (provided $\mathcal{P} \neq \text{NP}$) there is no absolute approximation algorithm for the Graph Steiner Tree Problem.

(2 points)

Please turn in your solutions on Tuesday, June 9th, before the lecture.