Summer Term 2015
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Approximation Algorithms

Exercise Sheet 4

Exercise 4.1:
Consider the Directed Steiner Tree Problem: Given an edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at r that contains every vertex in T.
Show that a k-approximation algorithm for the Directed Steiner Tree Problem can be used to obtain a k-approximation algorithm for Set Cover.

(2 points)

Exercise 4.2:
An instance of Maximum Satisfiability is called k-satisfiable if any k of its clauses can be satisfied simultaneously. Give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a $\frac{\sqrt{5}-1}{2}$-fraction of the clauses.

(4 points)

Exercise 4.3:
Describe an algorithm which decides if an undirected graph $G = (V, E)$ is 4-colorable in time $\mathcal{O}(|E| \cdot 2^{|V|})$.

(2 points)

Exercise 4.4:
Show that any 4-colorable graph with n vertices can be colored with $\mathcal{O}(n^{2/3})$ colors in polynomial time.

(4 points)

Exercise 4.5:
Describe exact algorithms with running times of $\mathcal{O}(2^{n/2})$ for the following problems:

(i) Subset Sum, where n is the number of numbers.

(ii) Knapsack, where n is the number of items.

(4 points)

Please turn in your solutions on Tuesday, **May 12th**, before the lecture.