Summer Term 2015
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Approximation Algorithms

Exercise Sheet 3

Exercise 3.1:
Show that the following problem is NP-hard: Given an instance of 2SAT, compute a truth assignment satisfying the maximum number of clauses.

(4 points)

Exercise 3.2:
Consider the following procedure for the unweighted MINIMUM VERTEX COVER PROBLEM: Given a connected undirected graph G, choose any vertex $r \in V$ and compute a directed DFS tree T rooted at r. Return all vertices with non-zero out-degree in T. Show that this is a 2-approximation algorithm.

(3 points)

Exercise 3.3:
The LP relaxation of the MINIMUM WEIGHT VERTEX COVER PROBLEM is

$$\min\{cx : M^T x \geq 1, x \geq 0\},$$

where $M \in \{0,1\}^{n\times m}$ is the incidence matrix of an undirected graph G and $c \in \mathbb{R}^n_+$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

(i) Show that the above LP relaxation always has a half-integral optimum solution.

(ii) Use (i) to obtain a 2-approximation algorithm for the MINIMUM WEIGHT VERTEX COVER PROBLEM. Is the analysis tight?

(3+1 points)

Exercise 3.4:

(i) Prove that the following is a 2-approximation algorithm for the k-CENTER PROBLEM: Given an instance (X, dist, k), start with $C = \{x\}$ for $x \in X$ arbitrary. Successively add some $x \in X \setminus C$ to C maximizing $\min_{c\in C} \mathit{dist}(x, c)$ until $|C| = k$. Return C.

(ii) Show that there is no α-approximation algorithm (provided that $P \neq NP$) for the k-CENTER PROBLEM if we do not require the distance function to satisfy the triangle inequality, where α is any polynomially computable function in the instance size.

(3+2 points)

Please turn in your solutions on Tuesday, **May 5th**, before the lecture.