Summer Term 2015 | Research Institute for Discrete Mathematics
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Approximation Algorithms

Exercise Sheet 10

Algorithm 1: Steiner Tree Approximation

Input: $G = (V, E)$, $R \subseteq V$, $c \colon E \to \mathbb{R}_{>0}$
Output: a Steiner tree T

$I := \emptyset$
repeat
  foreach $v \in V \setminus (R \cup I)$ **do**
    $I' := I \cup \{v\}$
    remove vertices from I' having degree 1 or 2 in $MST(R \cup I')$
    if $mst(R \cup I') < mst(R \cup I)$ **then**
      $I := I'$
until *no improvement during last iteration*
return a Steiner tree corresponding to the MST computed in the last iteration

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Exercise 10.1:
Consider the Steiner Tree Problem restricted to instances for which $V \setminus R$ forms a stable set. Give a tight analysis of the approximation algorithm for this problem described by Algorithm 1, i.e., determine the number $\alpha \geq 1$ such that

(i) Algorithm 1 is an α-approximation for this restriction of the Steiner Tree Problem and

(ii) there is a class of instances on which the algorithm does not necessarily perform better than an α-approximation.

Here, we denote by $MST(R)$ a minimum spanning tree in the terminal distance graph for the vertex set R and by $mst(R)$ its length.

(4 points)

Exercise 10.2:
Consider the relative greedy algorithm.

(i) For every $k \in \mathbb{N}$ describe an instance for which the relative greedy algorithm does not necessarily find an optimum solution.

(ii) Can you improve the approximation guarantee of the algorithm for $k = 5$?

(2 + 1 points)

Exercise 10.3:
Consider the restriction of the TRAVELING SALESMAN PROBLEM to complete graphs in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for this problem.

Hint: Compute a minimum-weight perfect 2-matching.

(3 points)

Exercise 10.4:
Consider the following algorithm for the SYMMETRIC TRAVELING SALESMAN PROBLEM with triangle inequality:
Consider an instance (K_n, c). Start with an arbitrary node $u \in V(K_n)$. Find a shortest edge $e = \{u, v\} \in E(K_n)$ connecting u to another node v. This yields a subtour $T = (u, v, u)$. Let $U := V \setminus \{u, v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in T.

(ii) Add w to T between two neighboring nodes $i, j \in T$ (by deleting edge $\{i, j\}$ and connecting i and j with w) such that the cost of the new tour is minimized, i.e., find neighbouring $i, j \in T$ such that that $d(i, w) + d(w, j) - d(i, j)$ is minimum. Remove w from U afterwards.

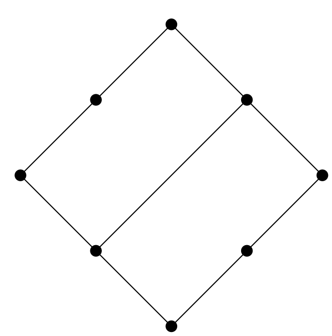
Show that this is a 2-approximation algorithm.

(3 points)

Exercise 10.5:
Show that the following problem is NP-complete: Given a graph G and a Hamiltonian circuit $C \subseteq E(G)$, is there another Hamiltonian circuit $C' \subseteq E(G)$?

Hint: Use the gadget below.

(3 points)



Please turn in your solutions on Tuesday, **June 30th**, before the lecture.